A Q-Theory of Banks*

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Bank capital requirements are based on book values, which are slow to reflect losses. In this paper, we develop a dynamic model of banks to study the interaction of regulation and delayed accounting. Our model explains four stylized facts: book and market values diverge during crises, the market-to-book ratio predicts future profitability, book leverage constraints rarely bind strictly even as market leverage fans out during crises, and banks delever gradually after net-worth shocks. We show how delayed accounting can allow the regulator to achieve better outcomes than immediate (mark-to-market) accounting. In an estimated version of the model, the optimal regulation couples faster loan-loss recognition with a modest relaxation of the book leverage constraint.

Keywords: Bank Leverage Dynamics, Market vs. Book Values, Accounting Rules, Bank Regulation, Financial Stability

JEL: G21, G32, G33, E44

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1 Introduction

Financial markets are quick to reflect bank equity losses. In contrast, book values are slow to recognize such losses. This difference in the speed of loss recognition leads to striking disparities between the behavior of banks' market equity and their book equity, which is reflected in Tobin's Q, the market-to-book equity ratio.[1] These differences are particularly accentuated during crises. For instance, during the 2007-2008 financial crisis—the most severe since the Great Depression—the aggregate book equity of banks remained stable while market values were eroded. Similarly, during the 2023 regional banking crisis, Silicon Valley Bank appeared well-capitalized on its books while abnormally leveraged in market value terms. Financial regulation is designed to prevent such crises but is based on accounting values. This observation raises critical questions: How does regulation constrain bank risk-taking if it is based on delayed information? What are the implications of delayed information in accounting books for an ideal regulatory framework?

To address these questions, this paper introduces a dynamic banking model emphasizing the slow recognition of accounting losses. Delayed accounting is the source of a distinction between the "fundamental" value of equity, which fully incorporates information on losses, and the book value of equity, which does not. As in other models, liquidations occur when losses cause fundamental leverage to exceed a market-determined limit. A critical aspect of our theory is that regulation intended to prevent liquidations is written in terms of accounting books. This feature allows us to analyze how the speed of loss recognition impacts bank risk-taking behavior and the effectiveness of regulation. Our theory underscores the necessity of considering the limited information in accounting books to capture bank behavior accurately and guide regulatory design.

The model features risk-neutral banks that fund risky loans with deposits and internal equity. Exogenous, idiosyncratic loan default shocks lead to jumps in fundamental leverage and observable market-based leverage.[2] These shocks can provoke market-induced liquidations. Such liquidations are socially inefficient, as they entail restructuring costs. Moreover, these social losses are not privately internalized: deposits are priced risk-free due to implicit deposit insurance, and the bank's recovery value is independent of the magnitude of the loss that leads to a liquidation. As a result, banks take excessive risk, reaping the benefits of leveraged returns without internalizing the social costs.

In our framework, bank regulation aims to correct the market's inefficiency by limiting book-based leverage. However, unlike fundamental equity, which decreases immediately upon

[1] Specifically, throughout this paper, we refer to Tobin's Q as the market-to-book *equity* ratio rather than the market-to-book *asset* ratio.

[2] The fundamental equity value differs from the market equity value because only the inside equity owner (the banker) accesses lending opportunities, leading to a valuation premium for the outside equity investor. We capture this valuation differential through differing discount rates for bankers and outside equity owners.

realizing loan losses, book equity takes time to recognize these losses. As a result of this delay, regulation constrains fundamental leverage only as past losses are slowly recognized on the books.

Our model successfully explains four facts related to the Tobin's Q of publicly traded US banks.[3] First, the time series of banks' aggregate book equity and market equity diverge substantially, especially during crises. This is a phenomenon that many models, which do not explicitly distinguish between market and book measures, cannot capture.[4] Second, Tobin's Q reflects market values, which embed forward-looking information about future profitability and risks not captured in book values. This aligns with much of the accounting literature but contrasts with models assuming no differences between book and market measures.[5]

A third fact is the difference between the cross-sectional distribution of market and book leverage: The distribution of book leverage is stable over time, to the point that, even during the 2007-2008 financial crisis, only a minor fraction of banks violated their regulatory capital ratios. By contrast, the dispersion of market-based leverage is highly volatile and rose dramatically during that period.

Finally, our fourth fact is the slow market and book leverage dynamics after net-worth shocks. We identify net-worth shocks by exploiting cross-sectional variation in banks' excess stock returns, using a factor model that partials out risk premia variation. In particular, we estimate risk-adjusted return shocks for each bank-quarter data point and use these to construct impulse responses to net-worth shocks. After a negative net-worth shock, which mechanically increases market leverage on impact, banks reduce their market leverage slowly by reducing their liabilities, with minor adjustment on the equity side. In contrast, book equity declines gradually, consistent with our delayed accounting mechanism.

Our model explains these facts not only qualitatively but also quantitatively. We estimate its key parameters using a simulated method of moments that targets the cross-sectional facts. We estimate the discount rates of investors and bankers, the size of loan default shocks, the market-based leverage constraint, and, importantly, the speed of loan loss recognition, α, a parameter whose value we modify to study policy counterfactuals regarding accounting rules. In particular, α is identified from the mean-reversion in the impulse responses of market leverage and liabilities to excess-return shocks. Our estimates also align with the

[3]In the Appendix, we show to what extent these facts are different for non-financial firms. Notably, non-financial firms have much lower leverage and are not constrained by regulatory capital ratios regarding book values.

[4]Papers that study the asset pricing implications of intermediary net worth (e.g., He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014) use market equity as a state variable. Papers focusing on the effects of regulation use book equity measures (e.g., Adrian and Boyarchenko, 2013; Begenau, 2020; Adrian and Shin, 2013; Corbae and D'Erasmo, 2021; Begenau and Landvoigt, 2022). The discrepancy between both measures has led to a debate on the best way to model banks (e.g., Adrian, Etula and Muir (2014) and He, Kelly and Manela (2017)). We argue that it is important to incorporate both equity measures into the design of regulatory policies.

[5]Laux and Leuz (2010) document the flexibility of banks to account for losses.

time series of Tobin's Q and loan charge-offs.

A lesson from the paper is that because capital requirements are second-best instruments, regulators can exploit the loss-recognition speed as an additional policy tool. We capture this by allowing regulators to control α. This exercise reveals that the speed of loss recognition has costs and benefits. Delaying loss recognition can be beneficial because it allows banks hit by losses to postpone their deleveraging process, thereby reducing their need to decrease lending and alleviating the cost of stringent regulation. However, delayed accounting leaves room for banks with significant unrecognized losses to take excessive liquidation risk, potentially leading to more market-based liquidations. Because this additional tool introduces a trade-off, determining its optimal level becomes a quantitative question.

Naturally, the optimal loss-recognition speed should be determined jointly with capital requirements. We study an optimal once-and-for-all change in capital requirements and accounting rules, starting from the model's estimated steady state and considering transitional dynamics. We find that the optimal policy mix involves a slight relaxation of capital requirements from Basel III back to Basel II levels but, importantly, a substantial strengthening of accounting standards toward speedier loss recognition. The benefits of this policy change stem from lower bank liquidations. However, stricter accounting implicitly tightens effective capital requirements, so the model suggests accompanying that change with looser capital ratios. Thus, a first policy implication is that optimal *microprudential* policy should also emphasize better accounting standards and recognize their interaction with capital rules.

A second policy implication regards *macroprudential* policies. In particular, our model highlights an unintended effect of countercyclical capital buffers (CCyB). To study this macroprudential implication, we simulate a recession by introducing an aggregate shock that increases the frequency of loan losses. Surprisingly, a CCyB can increase liquidation risk, causing permanently lower lending after the recession is over. The reason is that temporarily relaxing capital requirements during a recession allows banks to increase their book leverage and, therefore, their fundamental leverage. As banks' equity risk and unrecognized losses scale with leverage, an increase in leverage can result in a wave of liquidations and depress lending through a reduction in aggregate bank equity.

As part of our investigation of macroprudential implications, we contrast the use of countercyclical capital requirements with countercyclical accounting standards. We demonstrate that relaxing accounting rules during a crisis is a better-targeted policy. A relaxation of accounting standards allows banks severely impacted by losses to postpone deleveraging while keeping capital requirements the same for unaffected banks. These policy implications suggest that accounting rules should be at the forefront of bank regulatory design.

Related Literature. Canonical macrofinance models usually employ one concept for equity.[6] Which concept is employed depends on the constraints that intermediaries face. Models motivated by agency frictions place constraints on market values (e.g., Jermann and Quadrini, 2012; Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2013).[7] Models with book-based constraints are motivated by questions related to regulation (e.g., Adrian and Boyarchenko, 2013; Begenau, 2020; Corbae and D'Erasmo, 2021; Elenev, Landvoigt and Van Nieuwerburgh, 2021; Bianchi and Bigio, 2022).

Relative to this literature, our paper makes three contributions: First, it synthesizes facts about banks' Tobin's Q, shedding light on how market and book equity constrain bank behavior. Second, we build a Q-theory of banks where both market and book equity matter. The theory explains market and book equity differences through a delayed accounting mechanism interacting with regulatory constraints.[8] Third, we conduct policy experiments focused on how reforms to regulatory accounting rules would impact banks and the effectiveness of capital regulation.

A large literature in banking and accounting studies the impact of accounting and regulatory rules on bank behavior.[9] One strand of that literature discusses what banking activities need to be reported on the bank's balance sheet.[10] Another strand of this literature focuses on how banking activities should be reported on the balance sheet and how it affects bank decisions.[11][12] Our paper contributes most directly to the second strand of this literature. It distinguishes itself from both by using a quantitative model to analyze the optimal combination of accounting rules and financial regulation.

Our normative and macroprudential analyses relate to the literature on accounting rules

[6] See, e.g., Kiyotaki and Moore (1997); Gertler and Kiyotaki (2010); Gertler and Karadi (2011); Gertler, Kiyotaki and Queralto (2012); Jermann and Quadrini (2012); He and Krishnamurthy (2012); Brunnermeier and Sannikov (2014); He and Krishnamurthy (2013); Gertler and Kiyotaki (2015); Gertler, Kiyotaki and Prestipino (2016); Nuño and Thomas (2017); Piazzesi, Rogers and Schneider (2022).

[7] Examples of such frictions include costly verification (Townsend, 1979; Bernanke and Gertler, 1989), lack of commitment (Hart and Moore, 1994), and moral hazard (Holmstrom and Tirole, 1997).

[8] Slow-moving bank leverage (Fact 4) can also be generated by other models (e.g., Brunnermeier and Sannikov, 2014; Gertler et al., 2016). The difference is that, in those models, the slow leverage dynamics follow from adjustment costs. The Q-theory in this paper delivers slow-moving leverage dynamics through delayed accounting, offering a microfoundation for adjustment costs in other models different from leverage-ratcheting incentives (DeMarzo and He, 2021) and debt overhang (Gomes, Jermann and Schmid, 2016).

[9] See Appendix A.2.2 for an overview of the bank accounting literature. Bushman (2016) and Acharya and Ryan (2016) offer a nice survey of the literature.

[10] This relates to the debate about how stricter regulations fueled shadow banking activities' rise after the GFC (e.g., Buchak, Matvos, Piskorski and Seru, 2018; Hachem and Song, 2021; Begenau and Landvoigt, 2022; Erel and Inozemtsev, 2024; Buchak, Matvos, Piskorski and Seru, 2024; Chernenko, Ialenti and Scharfstein, 2024).

[11] For example, several papers discuss the implications of delayed loss accounting incentives and their implications empirically (e.g., Peek and Rosengren, 2005; Caballero, Hoshi and Kashyap, 2008; Blattner, Farinha and Rebelo, 2023; Plosser and Santos, 2018; Flanagan and Purnanandam, 2019). Milbradt (2012) theoretically studies the effect of fair-value accounting rules on banks' trading behavior.

[12] Studies of the economic effects of zombie lending practices include Faria-e Castro, Paul and Sánchez (2024), Acharya, Lenzu and Wang (2021), and Acharya, Crosignani, Eisert and Eufinger (2024).

and their effects on financial stability and credit supply, where we find that accounting rules should optimally be set jointly with capital requirements.[13] Our model, however, abstracts from banks' ability to manipulate accounting rules to their advantage.[14] To our knowledge, our paper represents the first quantitative exploration of accounting rules and their interplay with regulatory capital constraints. The model here can be used as a framework for assessing both micro- and macroprudential impacts stemming from the implementation of new accounting standards, such as the Current Expected Credit Loss (CECL) accounting standard.[15]

Finally, our paper relates to a growing literature emphasizing bank heterogeneity as an important dimension of bank regulation (e.g., Corbae and D'Erasmo, 2021; Rios-Rull, Takamura and Terajima, 2023; Goldstein, Kopytov, Shen and Xiang, 2024; Begenau, Landvoigt and Elenev, 2024; Abad, Bigio, Garcia-Villegas, Marbet and Nuno, 2024). The novelty here is that we emphasize accounting standards as an important dimension of capital regulation beyond capital requirements. Our focus on additional dimensions to bank regulation is also shared with Corbae and Levine (2024), which investigates the role of competition.

2 Motivating Facts

In this section, we document four stylized facts that motivater our Q-theory.

Data. We construct a panel of banks using balance sheet and income statement data on US bank holding companies (BHCs) from the FR Y-9C regulatory reports filed with the Federal Reserve from 1990 Q3 to 2021 Q1. We merge the accounting data with market data from the Center for Research in Security Prices (CRSP). See Appendix A.1 for more details on our sample construction and additional results.

Motivating Fact 1: There Are Large Differences in Book Equity and Market Equity. Our first fact is that the banking sector's Tobin's Q—the ratio of market equity

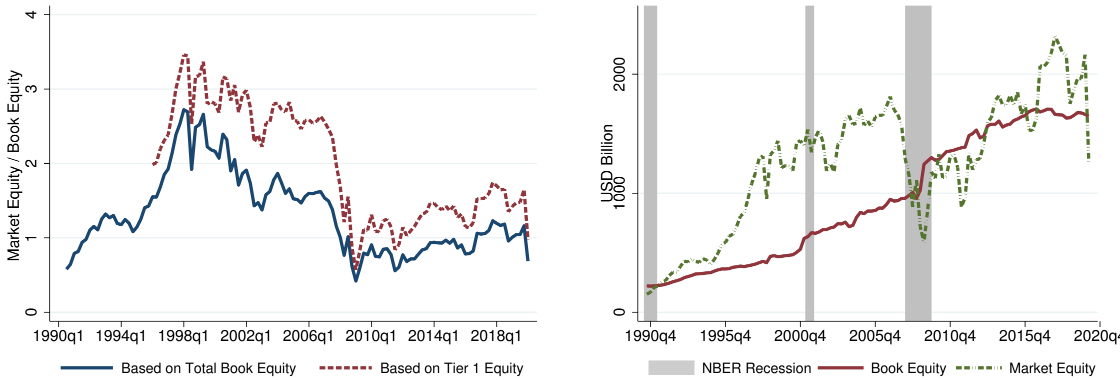
[13] In response to the financial crisis, the procyclical effects of mark-to-market assets were discussed (e.g., Shleifer and Vishny, 2011; Laux and Leuz, 2010; Plantin and Tirole, 2018). In response to the COVID-19 crisis, the extent of regulatory forbearance took center stage in macroprudential policy discussions (see Blank, Hanson, Stein and Sunderam, 2020). Since the March 2023 banking crisis, there is renewed interest in accounting rules and their ability to conceal risk (e.g., Jiang, Matvos, Piskorski and Seru, 2023; Granja, 2023).

[14] Empirical investigations into the enforcement of accounting rules and the impact on bank lending practices appear in Agarwal, Lucca, Seru and Trebbi (2014) and Granja and Leuz (2018), among others. Behn, Haselmann and Vig (2022) and Haselmann, Sarkar, Singla and Vig (2022) study important political economy issues of financial regulation and adherence to accounting rules that we abstract from.

[15] For empirical evidence on how CECL accounting rules affect bank lending decisions, see Granja and Nagel (2023) and references therein.

Figure 1: Tobin's Q and Bank Equity Evolution

(a) Tobin's Q (b) Book and Market Equity



Notes: These figures show data on Tobin's Q in Panel (a) and book equity and market equity in Panel (b) for an aggregate sample of publicly traded BHCs. Tobin's Q is the ratio of market equity to book equity and the ratio of market equity to Tier 1 equity capital (only available since 1996). Book equity and Tier 1 equity are from the FR Y-9C. Market equity is from CRSP. Market equity equals shares outstanding times the share price, aggregated across publicly traded BHCs. All level variables are converted to 2012 Q1 dollars with the seasonally adjusted GDP deflator.

over book equity—fluctuates widely over time.[16] Panel (a) of Figure 1 shows the time series of Tobin's Q for the aggregate banking sector using two different book equity definitions: total book equity and Tier 1 equity.[17] Panel (b) shows the components of aggregate Tobin's Q across all BHCs. Market valuations often diverge from book valuations, especially during financial crises. For example, during the 2008/2009 financial turmoil, aggregate book equity looked unaffected by the crisis, in stark contrast to market equity, which significantly declined. By 2008 Q4, bank market equity had plummeted over 54% from its 2007 Q3 level, a steeper fall than the 42% drop in the S&P 500 index (numbers are adjusted for inflation with the seasonally adjusted GDP deflator).

Motivating Fact 2: Tobin's Q Predicts Cash Flows and Default Risk. Our second fact is that Tobin's Q predicts future cash flows, charge-offs, and distance to default (D2D) in the cross-section of banks, suggesting that the market equity value of banks contains information that book equity does not.

Figure 2 illustrates the cross-sectional relationship between banks' log market-to-book equity ratio and future cash flow, controlling for time fixed effects, the Tier 1 regulatory capital ratio, and log book equity.[18] Panel (a) demonstrates Tobin's Q as a predictor of next

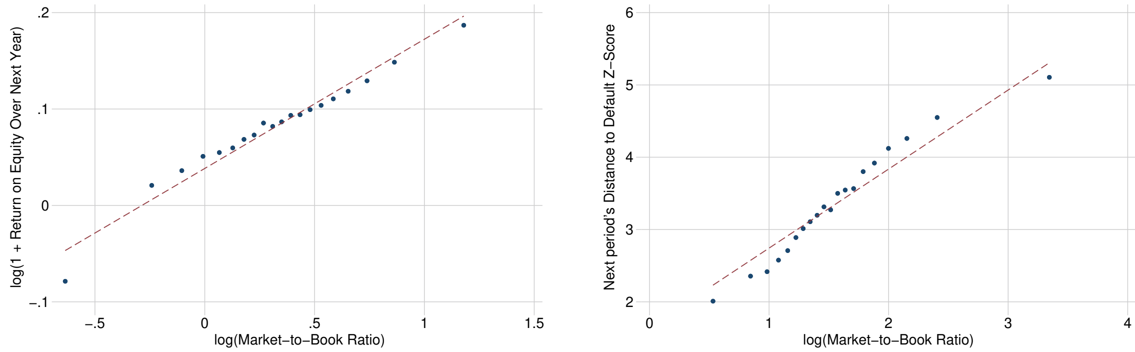
[16]Book and market equity differences during the GFC have been documented before (see, for example, Adrian and Shin, 2010; He et al., 2017). Our paper proposes a theory based on delayed loss accounting as a mechanism to explain the dynamics of bank Tobin's Q.

[17]While the former is available for our entire sample period, Tier 1 capital is a key variable for book regulatory constraints.

[18]To control for covariates, we residualize the left- and right-hand-side variables on the controls and then

Figure 2: More Cash Flow–Relevant Information in Market Than in Book Equity

(a) Return on Equity (b) Distance to Default



Notes: This figure presents cross-sectional binned scatter plots of log outcomes on the log Tobin's Q for BHCs. All plots include controls for log book equity, the Tier 1 capital ratio, and quarter fixed effects. Data on market equity are from CRSP. All other data are from the FR Y-9C reports. Return on equity over the next year is defined as book net income over the next four quarters divided by book equity in the current quarter. The Z-score distance-to-default measure over one quarter is calculated at the bank level as $\frac{\log(V/D)+\mu_V - \frac{1}{2}\sigma_V^2}{\sigma_V}$ (see Duffie, 2022). V denotes the total value of the bank measured as sum of the market value of equity and the book value of debt. D is measured as the book value of debt, or total liabilities. μ_A is the quarterly growth rate of V. σ_V is the standard deviation of the growth rate of V.

year's log return on equity (ROE), while Panel (b) links it to D2D. Banks with high Tobin's Q are on average further from default and more profitable over the next year. Additionally, Appendix Figure A.2 reveals that banks with high Tobin's Q generally have lower delinquent loan shares and lower future net charge-off rates. These findings suggest that variation in Tobin's Q stems partly from the differing informational content of book and market values.

Book measures are backward looking and record realized losses ex post, while market equity values capture current and future expected cash flows. Time series variation in Tobin's Q could also indicate discount rate movements, but this is unlikely in our cross-sectional analysis. Instead, the ability of Tobin's Q to forecast future accounting cash flows in the cross-section points to a delay in recognizing cash flow shocks in accounting values.

Motivating Fact 3: Regulatory Constraints Rarely Bind Strictly, and Market-Based Leverage Fans out During Crises. Our third stylized fact clarifies the nature of banks' leverage constraints. Panel (a) of Figure 3 presents the fraction of banks whose Tier 1 capital ratio falls below different cutoff values near the regulatory constraint; Appendix A.2.1 discusses how capital requirements have changed over time. The vast majority of banks keep a capital buffer above the required minimum. Even at the height of the financial crisis, over 90% of banks were "well capitalized" according to their Tier 1 capital ratio, and only 5% were below the regulatory minimum. Consistent with delayed recognition of loan losses,

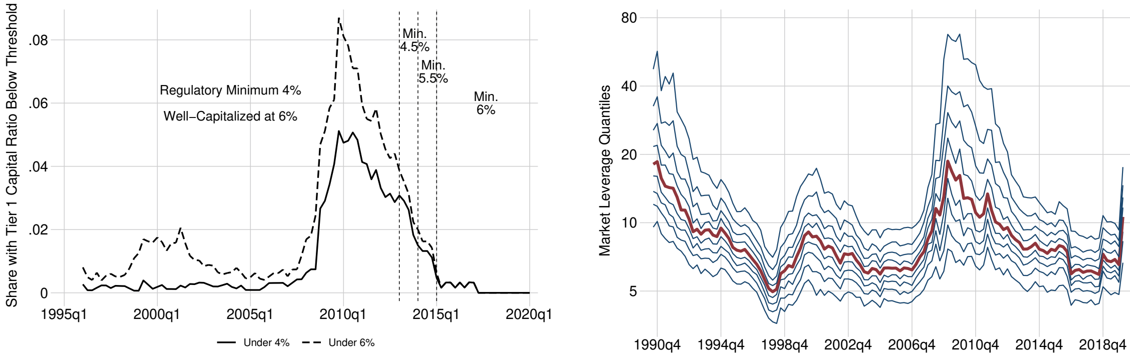
add back the mean of each variable to maintain the centering. Controlling log book equity is important to prevent spurious results due to ratio bias (see Kronmal, 1993).

the share of banks near the regulatory limit peaked only by the first quarter of 2010, two years after the crisis began.

Figure 3: Leverage Constraints

(a) Regulatory Constraints

(b) Market Leverage Quantiles



Notes: This figure shows the distribution of bank holding companies constrained by capital requirements in Panel (a) and the quantiles of market leverage in Panel (b) for BHCs on a log scale. Panel (a) plots the share of banks whose regulatory Tier 1 capital ratio, defined as (Tier 1 Capital)/(Risk-Weighted Assets), falls below a given threshold, computed from the full, unweighted sample. The regulatory capital requirements are shown on the graph and described in Appendix A.2.1. Book data (liabilities) come from the FR Y-9C, and market equity data are from CRSP. In Panel (b), market leverage is computed as (Liabilities + Market Equity)/Market Equity. The median value is plotted in bold red. Each tenth percentile is plotted in the thin blue lines.

Panel (b) of Figure 3 plots quantiles of the market leverage distribution over time. The median is plotted as a bold red line, and other deciles are plotted as thinner blue lines. Market leverage rose during each episode of banking stress: during the savings and loan crisis (1990–1991), during the financial crisis (2008–2009), and at the onset of the COVID-19 pandemic (2020–2021). The cross-sectional distribution of market leverage widened during these episodes. Between 2006 Q4 and 2009 Q1, the 90th percentile of market leverage rose nearly eightfold, from 8.5 to 67, while the median percentile rose only from 5.2 to 17.7. This pattern is inconsistent with binding market leverage constraints during a crisis. If market leverage constraints were binding, we would expect a compression of the market leverage distribution as more banks hit the constraint. However, Panel (b) shows an increase in this dispersion, in contrast to the expected compression due to bunching at the constraint.[19]

The distribution of bank leverage differs notably from that of nonfinancial firms. As shown in Appendix Figure A.7, market and book leverage are much lower and less dispersed for nonfinancial firms than for banks. The literature has rationalized high bank leverage as a result of deposits providing liquidity services, as well as government guarantees that implicitly subsidize bank deposits. Banks' incentive to carry high leverage, potentially in

[19]Figure A.3 in Appendix A.2.4 shows the distribution of book leverage over time: it is much less dispersed and more stable than the market leverage distribution.

excess of the social optimum, and regulatory constraints on bank leverage are two critical components that distinguish our model of banks in Section 3 from models of nonfinancial firms.

Motivating Fact 4: Leverage Dynamics Are Slow. Our fourth and final fact is that banks adjust slowly in response to idiosyncratic shocks. Shocks create a persistent gap between market and book equity, impacting Tobin's Q. Market leverage is also slow to revert to pre-shock levels, with the adjustment driven primarily by a change in liabilities rather than a recovery in market equity. We show this empirically using a distributed-lag model (e.g., Kilian, 2009) to represent changes in the bank's outcome variable $y_{i,t}$ as a function of a net-worth shock $\varepsilon_{i,t}$:

$$\Delta \log(y_{i,t}) = \alpha_t + \sum_{h=0}^{k} \beta_h \cdot \varepsilon_{i,t-h} + \psi_{i,t}, \tag{1}$$

where i indexes banks, t indexes quarters, k is the estimation horizon,[20] the outcome is $\Delta \log(y_{i,t}) = \log(y_{i,t}) - \log(y_{i,t-1})$, α_t is a time fixed effect, $\varepsilon_{i,t}$ denotes the mean-zero net-worth shock (defined in the next paragraph), and $\psi_{i,t}$ is an estimation error term. Given a shock $\varepsilon_{i,t}$, Equation (1) allows us to construct impulse-response functions (IRFs) for Tobin's Q and other bank outcome variables of interest. By including time-fixed effects, we isolate idiosyncratic from aggregate shocks and recover partial-equilibrium IRFs estimated from the cross-sectional variation in shocks. To report the IRFs, we sum the coefficients cumulatively to trace the response to a unit shock in $\varepsilon_{i,t}$. That is, the IRF is defined as

$$\mathrm{E}_t \left[\log(y_{i,t+k}) \,|\, \varepsilon_{i,t} = 1\right] - \mathrm{E}_t \left[\log(y_{i,t+k})\right] = \sum_{h=0}^{k} \beta_h.$$

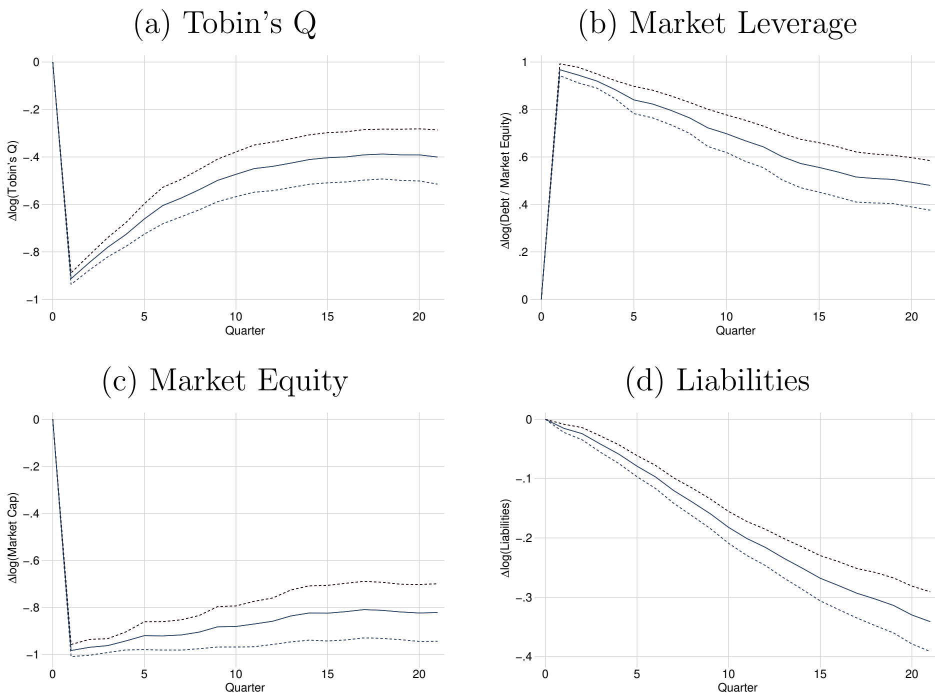
We interpret the shocks, $\varepsilon_{i,t}$, as idiosyncratic shocks to the bank's net worth, reflecting changes in expected cash flows. In our model, we show that these shocks follow from loan defaults. To estimate these net-worth shocks, we use shocks to banks' excess stock returns—see Appendix Section B.1. The main idea is based on the efficient-markets hypothesis: after adjustment for risk-premia, excess returns are ex ante unpredictable. Cross-sectional variation in $\varepsilon_{i,t}$ then represents unanticipated shocks that perturb bank equity. Our main empirical challenge is to identify these shocks empirically, $\varepsilon_{i,t}$, and isolate them from shocks to (a) the discount rate (risk premia) or (b) future investment opportunities.

To remove discount rate shocks, we decompose each bank's log excess stock return into an idiosyncratic component and a factor component by estimating a five-factor model for

[20] In all specifications, we set $k = 20$.

each bank as in Gandhi and Lustig (2015).[21] This isolates idiosyncratic, risk-adjusted return shocks for each bank, akin to the procedure in Vuolteenaho (2002). We then use these estimated return shocks, $\hat{\varepsilon}_{i,t}$, as instruments for the bank's log stock returns, in a model analogous to Eq. (1).[22] We conduct various robustness checks to validate our identification strategy in Appendix Section B.3.

Figure 4: Estimated Impulse Responses



Notes: These figures show the estimated percent impulse responses to a 1% negative return shock. The y-axis of our plots shows the contemporaneous response ($-\beta_0$) as quarter 1, the cumulative response one quarter later ($-\beta_0 - \beta_1$) as quarter 2, and so on. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The panels display the impulse responses of log Tobin's Q in Panel (a), log market leverage in Panel (b), log market equity in Panel (c), and log liabilities in Panel (d). Market leverage is defined as (Liabilities/Market Capitalization). The sample is publicly traded BHCs from 1990Q3 to 2021Q1 using FR-Y-9C and CRSP data.

Figure 4 presents the IRFs to a negative 1% return shock. The key takeaway is that banks adjust very slowly. Panel (a) shows that a 1% negative return shock lowers Tobin's Q by approximately 0.9% on impact, with a partial recovery over the next four years. The shock affects the components of Tobin's Q, market equity and book equity, differently. In Panel (c), market equity falls immediately by approximately 1% on impact and recovers to -0.8% after four years, remaining stable thereafter. Book equity declines slowly, reaching -0.5% only after 10 quarters (see Panel (f) in Appendix Figure A.4). These results imply that net worth shocks that are immediately recognized in market equity are only slowly recognized on

[21] These five factors are the three Fama–French factors (Fama and French, 1993), a credit factor calculated as the excess return on an index of investment-grade corporate bonds, and an interest rate factor calculated as the excess return on an index of 10-year US Treasury bonds.

[22] In Appendix Section B.1, we prove that this consistently estimates the coefficients of the true model.

banks' books. The responses of bank market leverage and liabilities in Panels (b) and (d) also suggest a slow adjustment process. In response to a negative net-worth shock, banks delever by slowly paying off liabilities. In sum, cash flow shocks drive a long-lasting wedge between the market and book valuations of banks and also drive gradual adjustment dynamics in leverage.

In Appendix Section B.4, we examine heterogeneity in impulse response functions across banks. We do not find robust evidence of heterogeneous impulse responses; however, we have limited statistical power to pick up these differences.

Appendix A.3 presents the stylized facts using data from publicly traded nonfinancial firms. We find that some of the facts (Facts 2 and 4) are similar among nonfinancial firms, whereas others (Facts 1 and 3) are not. The fact that accounting values are slow to incorporate losses in nonfinancial firms is not surprising given that delayed loss accounting rules affect nonfinancial firms as well. The key difference between banks and nonfinancials that we focus on is the fact that banks are much more debt-financed than nonfinancials and that banks face regulatory constraints in terms of book values, inducing an *interaction* between book-based accounting rules and regulatory leverage constraints.

3 Q-Theory

This section presents our Q-theory of banks. We embed this banking block into a general equilibrium setting when we discuss the normative implications. Proofs and further details are found in the appendix.

3.1 Model

Environment. Time is indexed by $t \in [0, \infty)$. All assets are real. A continuum of banks with unit mass funds loans, $L \geq 0$, with deposits, $D \geq 0$, and equity, $W \equiv L - D$. The demand for loans and supply of deposits are perfectly elastic at the rates r^L and r^D, respectively.

Bank Objective. Each bank maximizes the expected discounted value of future dividends:

$$V_0 = \mathbb{E}\left[\int_0^\infty \exp\left(-\rho t\right) C_t dt\right],$$

where C_t denotes dividends at instant t and $\rho > 0$ is the discount rate. Banks follow a constant dividend rate rule, $C_t = cW_t$. In the quantitative section, banks choose the dividend rate, but this is an inessential feature introduced only for calibration purposes.

Loan Defaults and Portfolio Decision. Loan defaults are the only risk banks face. Defaults are i.i.d. across banks and arrive according to a right-continuous (or càdlàg) Poisson process dN, with intensity σ. When the shock arrives, a fraction ε of L defaults.

At each instant, banks choose leverage $\lambda \geq 1$, with $L = \lambda W$ and $D = (\lambda - 1) W$. Equity satisfies the following stochastic-differential equation:

$$dW = \underbrace{\left[\underbrace{r^L \lambda - r^D (\lambda - 1)}_{\text{ROE}} - c \right] W}_{\equiv \mu^W W} dt - \underbrace{\underbrace{\varepsilon \lambda W}_{\text{default loss}}}_{\equiv J^W W} dN. \tag{2}$$

Equity features a scaled drift, μ^W, which increases with the ROE and decreases with the dividend rate c. Equity losses occur upon a default, appearing in the scaled equity jump term J^W. Importantly, equity losses scale with λ.

Notation. For a level variable x, we use $\mu^x W$ to denote the drifts scaled by equity and $J^x W$ to refer to its jump scaled by equity. When a variable x is a ratio, the scaling is unnecessary—μ^x and J^x denote unscaled drifts and jumps. Below, we distinguish between book and fundamental values, denoting by $\bar{X}$ the book value of fundamental variable X. We use calligraphic letters to represent economy-wide aggregates: e.g., $\mathcal{L}$ represents the aggregate stock of loans.

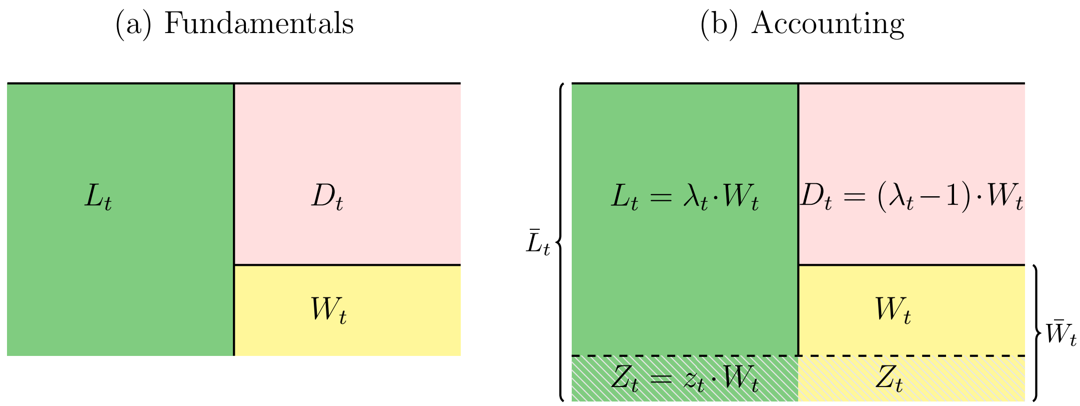
Equity Definitions, Accounting Rules, and Zombie Loans. We distinguish between three forms of equity: the fundamental value encountered above, W; the book (or accounting) value, $\bar{W}$; and the market value, S. Book equity $\bar{W}$ differs from fundamental equity W. Whereas fundamental equity immediately reflects defaults, book values record losses with a lag.

Book equity is relevant because regulation is based on accounting books. The gap between fundamental and book equity is given by the stock of unrecognized defaults Z, which we call zombie loans. On the books, loans appear as the sum of fundamental loans plus zombie loans Z, $\bar{L} \equiv L + Z$. Thus, book equity includes zombie loans as well, $\bar{W} \equiv \bar{L} - D = W + Z$. We label the stock of unrecognized losses as zombie loans because they survive as loans on the accounting books but are "dead" in the sense that they will not yield income going forward.

Figure 5 sketches the bank's fundamental balance sheet in Panel (a) and accounting balance sheet in Panel (b). Zombie loans are part of the stock of book loans and book equity. We define the zombie loan ratio, $z \equiv Z/W$, as the ratio of zombie loans to fundamental equity. Book leverage is $\bar{\lambda} \equiv \bar{L}/\bar{W} = (\lambda + z) / (1 + z)$. As is clear from the figure, banks seem less levered on the books when the zombie ratio is higher.[23]

[23] $\lambda - \bar{\lambda} = (\lambda - 1) z / (1 - z) \geq 0$ is increasing in z.

Figure 5: Fundamental and Accounting Balance Sheet



Notes: This figure shows the balance sheet of the bank in terms of fundamental values (Panel (a)) and book values (Panel (b)).

While fundamental and book equity differ by the amount of zombie loans, fundamental equity W and market equity S differ when shareholders' discount rate ρ^I differs from the return on equity.[24] We articulate a notion of market-based equity to decompose Tobin's Q, defined in the usual sense as $Q \equiv S/\bar{W}$, into the product of two ratios:

$$Q \equiv \frac{S}{W} \cdot q, \tag{3}$$

the market-to-fundamental equity ratio S/W and the fundamental-to-book equity ratio, or little $q \equiv W/\bar{W} = 1/\left(1+z\right)$, the novel feature of our theory. We use our measure of market-based equity to construct model-based excess stock-return shocks analogous to those in the data. Whereas book equity and market equity have data counterparts; the fundamental value does not.

Informational Assumptions and Timing. Banks face the possibility of market and regulatory liquidations. Market discipline induces liquidations if fundamental leverage λ exceeds an upper bound κ. Regulators liquidate banks if their book leverage exceeds a regulatory limit Ξ. Thus, banks are liquidated if at any instant t they violate either of the following constraints:

$$L/W \leq \kappa \quad \text{or} \quad \bar{L}/\bar{W} \leq \Xi. \tag{4}$$

If banks are liquidated, bankers recover an exogenous fraction v_0 of the fundamental equity. A liquidated bank is replaced by a new bank that starts with $z = 0$ and the remaining equity of the liquidated bank.

[24]The shareholder values the bank based on its stream of dividend payments. If leverage, and thus the return on equity, are constant, then this yields the valuation $\frac{c}{\rho^I - (\text{ROE} - c)} W$ via the Gordon growth formula. This will equal W only if $\text{ROE} = \rho^I$.

We combine the inequalities in (4) into a single constraint in terms of λ and z:

$$\lambda \leq \Gamma(z) \equiv \min\{\kappa,\ \Xi + (\Xi - 1) z\}. \tag{5}$$

We label $\Gamma(z)$ the *liquidation boundary.* If at any instant $\lambda > \Gamma(z)$, the bank is liquidated. Regulation is more stringent than the market-based constraint if $z < z^m \equiv (\kappa - \Xi) / (\Xi - 1)$. Note that a higher zombie ratio decreases book leverage, therefore effectively relaxing the constraint in terms of fundamental leverage.

Banks never choose to be liquidated voluntarily by setting their leverage above the liquidation boundary. However, banks face involuntary liquidations because their loans are risky, and they do not control their leverage at the moment of a loan default. Hence, it is important to understand how the variables in constraint (5) jump when a default event occurs; liquidations are triggered when these variables jump to a point where the constraint is violated.

We assume that investors have real-time information on the bank's fundamental and accounting variables. Thus, they are perfectly informed about the state variables of the bank, W and Z. Hence, at the moment of a default event, a bank with leverage λ is liquidated by market discipline if the following condition is violated:

$$\lambda + J^{\lambda} \equiv \frac{\overbrace{L - \varepsilon L}^{\text{loans after default}}}{\underbrace{W - \varepsilon L}_{\text{equity after default}}} = \frac{\lambda(1-\varepsilon)}{1 - \varepsilon\lambda} \leq \kappa. \tag{6}$$

Because fundamental leverage jumps to $\lambda + J^{\lambda}$ at the moment of a default, leverage may violate (5). If the bank survives the default episode; it can reverse the jump in leverage by selling part of its loans immediately after the event.

Market-induced liquidations occur because banks cannot immediately offset the jump in leverage by selling assets. In technical terms, leverage is non-adapted—it jumps at the instant of a default event and then reverts. This is the analog of a discrete-time setting where leverage is a beginning-of-period choice, but a random shock at the end of the period alters its value.

The public nature of market prices implies that regulators can infer W and Z. Therefore, markets and regulators share the same information set. Critically, however, regulators cannot enforce regulation on the basis of market values, even though they can perfectly infer Z from market values. Regulation can lead to bank liquidations only if bank accounting values show proof of regulatory noncompliance.

While banks can hide losses on their books, they still face the risk of regulatory liquidations. This is because banks cannot hide losses instantaneously. We assume that, at the moment of default, regulators can use the equity loss $\varepsilon\lambda W$ as evidence if they intervene. We assume that regulators intervene in a bank and demonstrate a lack of compliance whenever the bank is provably violating the regulatory limit. Thus, banks are liquidated by the regulator if the following condition is violated:

$$\bar{\lambda} + J^{\bar{\lambda}} \equiv \frac{\overbrace{\bar{L} - \varepsilon L}^{\text{book loans after default}}}{\underbrace{\bar{W} - \varepsilon L}_{\text{book equity after default}}} = \frac{\lambda(1-\varepsilon) + z}{1 - \varepsilon\lambda + z} \leq \Xi. \tag{7}$$

If the bank survives the default event without regulatory liquidation, it can conceal its loss by adding it to the stock of zombie loans the instant after a default event. Once these losses are concealed as zombie loans, regulators cannot use them as evidence of noncompliance. Moreover, since hiding losses relaxes the bank's constraints in the future, the bank will always choose to hide losses. Thus, zombie loans jump immediately after each default event survived by the bank. As a result, Z_t is also a non-adapted process. We discuss the motivation for our informational assumptions in greater detail below.

If regulators were oblivious to bank losses, Equation (7) would not include default losses and Z_t would be adapted. As we show in Appendix E.1, banks would face only the risk of market-based liquidations but would not be affected by the regulatory constraint and would not keep a regulatory buffer. We describe the timing and stochastic processes corresponding to each model variable in greater detail in Appendix D.1.

Shadow Boundary. The *shadow boundary* is a key object. For a given z, the shadow boundary $\Lambda(z)$ is the maximum leverage such that the bank survives a default shock:

Lemma 1 *[Shadow Boundary] A bank satisfies the survival conditions (6)–(7) if and only if $\lambda \leq \Lambda(z)$ where:*

$$\Lambda(z) = \min\left\{\frac{\Xi + (\Xi - 1)z}{1 + (\Xi - 1)\varepsilon}, \frac{\kappa}{1 + (\kappa - 1)\varepsilon}\right\}.$$

$\Lambda(z) = \kappa/(1 + (\kappa - 1)\varepsilon)$ *when* $z > z^s \equiv \frac{1-\varepsilon}{1-\varepsilon+\varepsilon\kappa} \times \frac{\kappa-\Xi}{\Xi-1} = \frac{1-\varepsilon}{1-\varepsilon+\varepsilon\kappa} \times z^m$.

The formula for $\Lambda(z)$ shows that, for $z \leq z^s$, a larger z allows banks to lever up safely, avoiding regulatory liquidations. When $z > z^s$, the shadow boundary is flat because the market-based constraint is the relevant margin. Figure 6 depicts an example of a pair of shadow and liquidation boundaries. We return to this figure to describe the dynamics when banks survive.

Evolution of Zombie Loans. Zombie loans evolve according to the left-continuous process:

$$dZ = \underbrace{\underbrace{-\alpha Z}_{\text{loss recognition rate}}}_{\equiv \mu^{Z} W} dt + \underbrace{\underbrace{\varepsilon \lambda W}_{\text{unrecognized default}}}_{\equiv J^{Z} W} dN, \tag{8}$$

where $\alpha > 0$ is meant to capture the speed of loan loss recognition. Zombie loans jump by the amount of losses the instant *after* default events. α reflects accounting rules and regulatory procedures that affect the speed of loan loss recognition. We pay special attention to how α governs the dynamics of bank variables, allowing us to match the data and show how it affects welfare.

The zombie loan ratio z has a law of motion:

$$dz = \underbrace{-z\left(\alpha + \mu^{W}\right)}_{\equiv \mu^{z}} dt + \underbrace{\lambda \varepsilon \left[\frac{1+z}{1-\lambda\varepsilon}\right]}_{\equiv J^{z}} dN. \tag{9}$$

z decreases faster with α and the equity growth rate μ^{W}, and jumps upon a default event.

Bank's Problem. The bank's state variables are $\{Z, W\}$, and it controls λ to solve a Hamilton–Jacobi–Bellman (HJB) equation.

Problem 1 *[Bank's Problem] The bank's optimal leverage $\lambda(Z, W)$ solves:*

$$\begin{aligned} \rho V(Z,W) &= \max_{\lambda \in [1, \Gamma(Z/W)]} cW + V_{Z}(Z,W)\,\mu^{Z} W + V_{W}(Z,W)\,\mu^{W} W \\ &+ \sigma \underbrace{\left[V\left(Z + J^{Z}, W + J^{W}\right) \mathbb{I}_{\lambda \leq \Lambda(Z/W)} + \upsilon_{o} W\, \mathbb{I}_{\lambda > \Lambda(Z/W)} - V(Z,W)\right]}_{\textit{jump in value after loan default}} \end{aligned} \tag{10}$$

subject to the law of motion of fundamental equity, (2), and the law of motion of zombie loans, (8).

Throughout the paper, the market value of equity, $S(Z, W)$ solves the same HJB equation, but with ρ^{I} replacing ρ, the bank's leverage choice taken as given, and assuming shareholders are wiped out when the bank is liquidated. For the rest of the paper, we assume:

Assumption 1

1. *Lending is profitable: $r^{L} - \sigma\varepsilon \geq r^{D}$.*
2. *Returns are bounded: $\rho > r^{D} + \left(r^{L} - r^{D}\right)\kappa - c$.*
3. *Liquidation is costly for self-financed banks: $\left(\rho - r^{L}\right)\upsilon_{o}/c \leq 1 - \varepsilon$.*

4. If indifferent between risking and not risking liquidation, the bank chooses to avoid liquidation.

The first condition guarantees that lending is profitable. The second condition bounds equity growth. The third guarantees that banks avoid liquidation for tight constraints but risk liquidations otherwise. The fourth condition implies that banks avoid risking liquidation unless they have a strictly positive benefit from doing otherwise.

Discussion of Model Assumptions. Our model incorporates several financial frictions: First, consistent with the intermediary asset pricing literature (He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014), bank equity can only grow through retained earnings. Second, depositors do not internalize bank liquidation risk in their pricing. This reflects implicit deposit insurance (Diamond and Dybvig, 1983) or government guarantees (Kelly, Lustig and Van Nieuwerburgh, 2016; Atkeson, d'Avernas, Eisfeldt and Weill, 2018). Finally, the solvency condition (5) forces liquidations and is shaped by market forces and regulatory considerations.

While recent banking models, such as Gertler and Kiyotaki (2015), impose constraints on market-valued leverage, our framework models market-based liquidations that depend on fundamental leverage. This approach builds on the rich corporate finance literature, e.g., Leland and Toft (1996), where once fundamental leverage exceeds a multiple, the firm is voluntarily closed. Deposit insurance and regulatory constraints tailor this model to the banking sector.

Delayed loss accounting, the novel feature, captures a bank's ability to engage in evergreening (Caballero, Hoshi and Kashyap, 2008) and avoid immediate recognition of market losses (Flanagan and Purnanandam, 2019). Banks create zombie loans by delaying charge-offs, avoiding reductions in regulatory capital. Since rolling over a loan does not require new funds, evergreening allows a bank to inflate its accounting equity. The effect is to relax the leverage constraint (5).

We further assume that bank equity prices contain information not in books. This aligns with the empirical findings in Section 2 suggesting that Tobin's Q contains more predictive power than book values. This information can be collected by analysts forecasting a bank's loan portfolio defaults based on sources other than the bank's books.

Our model makes two critical assumptions that shape the regulatory environment: First, regulators cannot close banks based on market data. To liquidate banks, regulators must intervene in the bank and build accounting-based evidence. This assumption is grounded in the legal constraints faced by regulators. Second, we assume that banks need time to conceal their losses. The assumption that losses cannot be hidden instantaneously reflects that evergreening requires time-consuming loan reprogramming. Likewise, moving assets from market to hold-to-maturity accounts takes time. As a result, the regulator only has a short window of opportunity to intervene upon the realization of default. Once losses are hidden, regulators cannot build a case for liquidation. The assumption also makes the bank vulnerable to regulatory liquidations if it takes excessive leverage.

The combination of both assumptions allows the model to feature both regulatory-liquidations risk and hidden losses parsimoniously. If regulators could never obtain evidence of bank losses, regulation would be immaterial. In turn, if regulators could constantly intervene banks there would be no scope for delayed accounting.

In principle, one could argue that regulators can constantly intervene banks to prevent them from hiding any losses. However, this argument would miss that regulatory interventions entail fiscal costs and unwarranted interventions risk provoking a regulatory backlash. This is why we assume that regulators intervene only when they know, possibly through market prices, that the bank violates the regulatory constraint. To elaborate on this point, in Appendix D.8, we present a simple sequential-form game based on costly state verification, in the spirit of Townsend (1979), consistent with this notion.[25] We contend that the informational and timing assumptions mirror the events of the regional banking crisis in 2023.[26]

The normative analysis introduces another friction: social liquidation costs, which are critical to justifying the need for regulation. Regulatory violations must lead to bank closures to ensure compliance. In our model, social liquidation costs are assumed to be identical for both market-based and regulatory liquidations. This assumption highlights why capital requirements are not a first-best instrument, paving the way for the policy tradeoffs analyzed in Section 5. Exploring optimal penalties for enforcement and distinguishing between the social costs of different liquidations lies beyond the scope of this paper.

3.2 Positive Analysis

We now present the solution to the bank's problem. We start with immediate accounting to explain the inherent risk-return tradeoff.

Immediate Loan Loss Recognition. Immediate loan loss recognition occurs when $\alpha \to \infty$ and $z_t = 0 \quad \forall t$. Consider the *laissez-faire regulation* where $\kappa < \Xi$. In this case, under immediate loss recognition, the shadow and liquidation boundaries simplify to constants, $\Gamma = \kappa$ and $\Lambda = \kappa \cdot (1 + \varepsilon (\kappa - 1))^{-1}$, respectively.

[25] Taking the $\Delta \to 0$ limit of this game provides a microfoundation for the timing of regulatory liquidations in the model. We thank Douglas Diamond for making a connection with these models.

[26] Notably, they mirror the dissolution of SVB in 2022 Q3: Despite being insolvent, that bank remained solvent under book-based regulatory standards. This disconnect resulted from held-to-maturity securities being valued at their amortized costs rather than their diminished market values. This discrepancy allowed a significant portion of SVB's securities holdings to appear inflated to meet regulatory standards. The regulator acted after the bank's stock valuation plummeted.

Proposition 1 *[Immediate Accounting Solution] With immediate accounting,* $V(0, W) = vW$ *and* $L = \lambda^* W$*, where* $v = c(\rho - (\Omega^* - c))^{-1}$*,* Ω^* *is the optimal expected levered equity return,*

$$\Omega^* = r^D + \max_{\lambda \in [1,\kappa]} \underbrace{\left(r^L - r^D\right)\lambda + \sigma\left\{(1 - \varepsilon\lambda)\,\mathbb{I}_{[\lambda \leq \Lambda]} + \frac{v_o}{v}\mathbb{I}_{[\lambda > \Lambda]} - 1\right\}}_{\text{portfolio objective } \equiv \Omega(\lambda)}, \tag{11}$$

and λ^* *is the optimal leverage in* Ω^**.*

This analysis yields three takeaways that carry through to the general case. First, the bank's problem scales with W. Second, the marginal value of bank equity, v, converts a unit of W into an anticipated net present value of dividends. Third, selecting the optimal leverage maximizes the expected return on equity Ω^*. This maximization balances a tradeoff between levered returns and liquidation risk.

If the bank sets $\lambda > \kappa$, it is immediately liquidated. Hence, $\lambda \in [1, \kappa]$. The objective function $\Omega(\lambda)$ in (11) is increasing in λ except at a discontinuous drop located at the shadow boundary Λ. This drop occurs because when leverage exceeds the shadow boundary, the bank risks liquidation.[27] Since the objective is piecewise linear, with a discontinuity, optimal leverage is either at the shadow or the liquidation boundary:

$$\Omega^* = r^D + \max\left\{\Lambda\left[\left(r^L - r^D\right) - \sigma\varepsilon\right], \left(r^L - r^D\right)\kappa - \sigma\left(1 - \frac{v_o}{v}\right)\right\}.$$

A parametric condition dictates which of the two corners is optimal.

Corollary 1 *Let* λ^o *be the unique (positive) solution to:*

$$\overbrace{\left(r^L - r^D\right)\left(\lambda^o - \frac{\lambda^o}{1 + \varepsilon(\lambda^o - 1)}\right)}^{\text{difference in levered return}} = \sigma\overbrace{\left(1 - \frac{v_o}{v} - \varepsilon\frac{\lambda^o}{1 + \varepsilon(\lambda^o - 1)}\right)}^{\text{difference in expected losses}}. \tag{12}$$

Optimal leverage is at the liquidation boundary, $\lambda^* = \kappa$*, if* $\kappa > \lambda^o$*. Otherwise, optimal leverage is at the shadow boundary,* $\lambda^* = \Lambda$*.*

The significance of the result is that banks risk liquidations, setting leverage to the liquidation boundary when leverage is permitted to be high enough. This is because the levered return scales with leverage, but liquidation recovery values are independent of leverage. If leverage is not permitted above a threshold, banks set their leverage to the shadow boundary, sacrificing returns but guaranteeing continuation.

Away from laissez faire, regulation is binding. In this case, with immediate accounting, the solution is isomorphic to the laissez-faire case, except that κ is replaced by the regulatory

[27] In Appendix D.5, we further discuss and plot the objective function in $\Omega(\lambda)$.

constraint Ξ. Thus, the optimal leverage, as outlined in Corollary 1, generalizes to:

$$\lambda^*(\Xi,\kappa)=\begin{cases}\min\{\kappa,\Xi\}\left(1+\varepsilon\left(\min\{\kappa,\Xi\}-1\right)\right)^{-1} & \text{if } \min\{\kappa,\Xi\}\leq\lambda^o\\ \\ \min\{\kappa,\Xi\} & \text{if } \min\{\kappa,\Xi\}>\lambda^o.\end{cases}\tag{13}$$

This bang-bang property carries through to the general case with delayed accounting.

Dynamics with Immediate Loss Recognition Accounting Rules. Under immediate loss recognition, the model has no internal propagation: banks instantly offset leverage changes via asset sales, leading to a single jump in the IRFs of total liabilities and book equity. Moreover, Tobin's Q is constant, as little q (the ratio of fundamental to book equity) is always one. Thus, Q lacks predictive power. Immediate accounting also eliminates cross-sectional variation in leverage ratios. The version of the model with immediate loan loss recognition is inconsistent with several of the facts presented in Section 2.

A common approach to producing variation in Tobin's Q and more sluggish adjustments of bank variables is to introduce balance sheet adjustment costs. Suppose we solved a variation of our model with adjustment costs and immediate accounting. This could generate a slow response of leverage, as in Fact 4, but would not cause losses to be predictable with Tobin's Q, as in Fact 2. This is because all losses would be immediately recorded on the books and so Tobin's Q should have no predictive power for ROE or charge-offs once we control for book equity. In an earlier version of this paper, we also estimated an alternative version with adjustment costs only. That model required implausibly high adjustment costs to explain the slow dynamics of leverage. Our delayed accounting mechanism provides a more plausible microfoundation for a "reduced-form" adjustment cost, producing slow-moving dynamics.

Delayed Accounting. We now characterize the solution under delayed accounting.

Proposition 2 *[General Solution] With delayed accounting, $V(Z,W)=v(z)W$ and $L(Z,W)=\lambda^*(z)W$ where:*

$$\rho v(z)=c-v_z(z)\alpha z+\left(v(z)-v_z(z)z\right)\cdot\left[\Omega^*(z)-c\right],\tag{14}$$

and $\lambda^*(z)$ solves:

$$\Omega^*(z)=r^D+\underbrace{\max_{\lambda\in[1,\Gamma(z)]}\left(r^L-r^D\right)\lambda+\sigma\left\{\frac{J^v(z,\lambda)}{v(z)-v_z(z)z}\right\}}_{\textit{portfolio objective}\ \equiv\Omega(z,\lambda)},$$

where $J^v(\lambda,z)\equiv v(z+J^z)(1-\varepsilon\lambda)\,\mathbb{I}_{[\lambda\leq\Lambda(z)]}+v_0\mathbb{I}_{[\lambda>\Lambda(z)]}-v(z)$.

With delayed accounting, the bank's problem is also scale-invariant: two banks with the same z behave as W-scaled replicas. The key difference is that z determines the shadow and liquidation values. For this reason, the valuation of equity $v(z)$ depends on z. The term $v(z) - v_z(z)z$ that multiplies the levered portfolio captures how an increase in equity increases the value of the bank directly and indirectly through z.

In this case, the choice of leverage depends on z, as shown next:

Corollary 2 *[Optimal Leverage] The optimal bank leverage, $\lambda^*(z)$, has the following bang-bang property:*

$$\lambda^*(z) = \begin{cases} \Lambda(z) & \text{if } \Omega(z, \Gamma(z)) \leq \Omega(z, \Lambda(z)) \\ \\ \Gamma(z) & \text{if } \Omega(z, \Gamma(z)) > \Omega(z, \Lambda(z)). \end{cases} \tag{15}$$

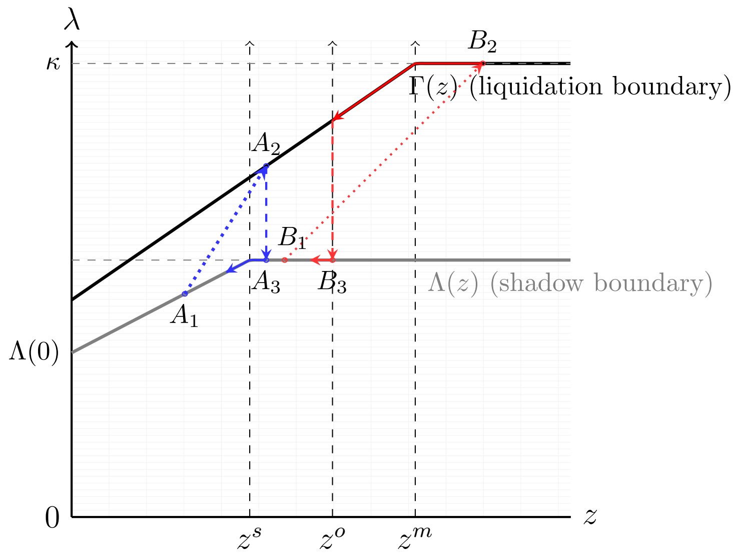
As with immediate accounting, under delayed accounting, leverage λ^* is a bang-bang control. It is at either the shadow or the liquidation boundary. The difference is that the solution depends on z. The intuition is the same. Banks risk liquidation when the returns Ω sufficiently counterbalance their liquidation risks. Since liquidation values are independent of leverage, banks risk liquidation when $\Omega(z, \Gamma(z)) > \Omega(z, \Lambda(z))$. As is common with bang-bang controls, leverage features discontinuities at points z^o such that $\Omega(z^o, \Gamma(z^o)) = \Omega(z^o, \Lambda(z^o))$.[28]

We use Figure 6 to explain the dynamics implied by Corollary 2. The points $\{A_1, A_2, A_3\}$ and $\{B_1, B_2, B_3\}$ are part of different trajectories in the $\{z, \lambda\}$-space. Consider a bank that receives a default shock starting at A_1. Before the shock, the bank sets leverage at the shadow boundary. In the absence of adjustment by the bank, the loan default would lead to a jump from A_1 to A_2: since the bank was on the shadow boundary before the shock, the shock puts it onto the liquidation boundary. The bank remains solvent. Because the zombie loan ratio jumps to a value below the discontinuity point z^o, the bank wants to avoid liquidation risk going forward. The bank sells loans to return to the shadow boundary, arriving at A_3 the instant after the shock. After the asset sale, the zombie loan and leverage ratios travel continuously along the shadow boundary as the books slowly recognize the loss.

If a loan default event occurs starting from B_1, the dynamics change. From that point, the shock pushed the bank to B_2 (note that since the bank starts to the right of z^s, where the market constraint is binding for the shadow boundary, the point it jumps to is to the right of z^m, where the market constraint binds for the liquidation boundary). Since the zombie loan ratio is higher than z^o, the bank opts to stay at the liquidation boundary, risking closure if it is hit by another shock. Provided that no further loan default events occur, the bank travels

[28] In general, there could be multiple such points, although in the estimated model, we find only one such discontinuity.

Figure 6: Typical Trajectories of z and λ under Delayed Accounting



Notes: These panels depict the shadow and liquidation boundaries and typical trajectories under delayed accounting, as characterized in Proposition 2. The dashed vertical lines correspond to z^s (the point at which the market constraint becomes binding for the shadow boundary), z^o (the level of the zombie loan ratio above which a bank chooses to stay at the liquidation boundary and risk closure), and z^m (the point at which the market constraint becomes binding for the liquidation boundary). The blue trajectory shows the path of a bank starting at A_1 after it is hit with a shock; because the shock does not push the bank past z^o, it immediately delevers back to the shadow boundary. The red trajectory shows the path of a bank starting at B_1: after it is hit with a shock, it stays at the liquidation boundary and only returns to the shadow boundary once its zombie loan ratio drifts down to z^o.

left along the liquidation boundary as books slowly recognize the loss. Once z reaches z^o, the bank chooses to delever to return to the shadow boundary at B_3.

Next, we turn to Section 4, where we demonstrate how an estimated version of the model can reproduce all four motivating facts. We can already anticipate why the model can explain the facts through Figure 6. Following a loan default event, book and market equity diverge due to the presence of zombie loans (Fact 1). Those zombie loans are gradually recognized on the books, making future book losses predictable (Fact 2). For appropriate parameters, most banks stay at the shadow boundary, maintaining a book leverage that is away from the liquidation boundary even if their market leverage is high (Fact 3). Because losses are recognized slowly, the bank delevers slowly in response to a negative shock (Fact 4).

4 Estimation and Matching Facts

This section describes how we map the model to the data and shows how it fits the facts from Section 2. More details are in Appendix Section F.

4.1 Model Parametrization

The model here is the same as that in Section 3, except we allow dividends to be a choice of the banker. We maintain risk neutrality but introduce a preference for smooth dividends: with risk neutrality, the bank's leverage choice is dictated by the tradeoff between returns and liquidation risk. Dividend smoothing allows us to match the IRFs more closely. To allow for dividend smoothing while keeping a risk-neutral objective, we endow the bank with Duffie-Epstein preferences with zero risk aversion and an intertemporal elasticity of substitution (IES) of $1/\theta$.[29] Recall that we assume that shareholders value bank equity differently from banks, which captures differences in the fundamental value of bank equity and the market value of bank equity.

We set $\left\{r^L, r^D, \Xi\right\}$ externally—see Appendix F—and jointly estimate $\left\{\rho, \rho^I, \theta, \varepsilon, \alpha, \kappa, \sigma, v_0\right\}$ via the simulated method of moments (SMM). We list the parameter values in Table 1. In Appendix Table 8, we show that the model matches targeted and untargeted moments well.

Table 1: Parametrization

Parameter	Description	Target
	Externally set parameters	
$r^L = 1.01\%$	Loan yield	BHC data: avg. interest income/loans
$r^D = 0.51\%$	Bank debt yield	BHC data: avg. interest expense/debt
$\Xi = 12.5$	Regulatory maximum asset to equity ratio	Capital requirement of 8%
	Jointly determined – estimated	
$\rho = 2.24\%$	Banker's discount rate	Book equity growth rate: 2%
$\rho^I = 3.47\%$	Investor's discount rate	Market-to-book ratio of equity: 1.316
$\theta = 7.94$	Banker's inverse IES	Market leverage IRF
$\varepsilon = 1.12\%$	Loan loss rate in event of default	Mean book leverage
$\alpha = 4.16\%$	Speed of loan loss recognition	Liabilities IRF
$\kappa = 51$	Market-based leverage constraint	Liabilities IRF
$\sigma = 0.115$	Arrival rate of loan default shocks	Mean quarterly net charge-off rate of 0.12%
$v_o = 0.046$	Bank liquidation value	Quarterly bank failure rate of 3.65 basis points

Notes: This table summarizes the parameter values, their role in the model, and the data target used to set or estimate their value. The text provides more details.

Jointly Determined Parameters. To produce model moment counterparts for each parameter draw, we simulate a quarterly panel from which we calculate the cross-sectional average moments and construct IRFs using the specification for the net-worth shock from Section 2. Our estimation targets the cross-sectional averages of book leverage, the book

[29] We show in Appendix G that calibrating $\theta = 2$ instead of estimating it slightly worsens the model's fit but does not qualitatively alter the predicted responses of the bank variables to loan default shocks.

equity growth rate, the market-to-book equity ratio, and the IRFs of bank liabilities and market leverage.[30] We require the model to match the charge-off rate and bank failure rate exactly.[31] We choose the arrival rate of the loan default shock to match a 0.12% quarterly net loan charge-off rate, setting $\sigma = 0.115$. We choose v_o, the banks' liquidation value, to match a quarterly bank failure rate of 3.65 basis points based on FDIC data.[32]

Identification and Estimated Values. The growth rate of book equity is informative about bankers' discount rate ρ because this parameter governs the dividend payout rate. In the data, the growth rate of book equity equals 2.00%; we estimate ρ to be 2.24%. To estimate investors' discount rate ρ^I, we target the average market-to-book ratio.[33] From Section 3.1, the market-to-book ratio of banks is $Q = s/(1 + z)$. We use the market-to-book ratio as a target since ρ^I enters the market valuation of banks s. We target an average market-to-book ratio of 1.316, which yields a value of $\rho^I = 3.47\%$.[34]

We use the IRF of market leverage as a target for θ.[35] Since dividends affect the market value of the bank, the IRF of market leverage to a net-worth shock is informative about θ. We estimate $\theta = 7.94$, which suggests a strong preference for near-constant dividend rates.

The distance between the shadow and the liquidation boundaries is determined by the loss size ε. Thus, given Ξ, the average book leverage ratio is informative about ε. We estimate $\varepsilon = 1.12\%$ to target an average book leverage ratio of 11.36.

We target the IRF of liabilities to identify κ and α. The loan loss recognition rate, α, governs how fast book equity reverts to fundamental equity. Recall that in response to a net-worth shock, fundamental leverage jumps and reverts with the reversion rate in z. Hence, the mean reversion in the IRF for liabilities $D = (\lambda - 1)W$ is informative about α, which

[30] Formally, the model is overidentified because each IRF in the data contains effectively 21 moments, one for each β_h in Eq. (1). In practical terms, these moments are highly correlated, so the de facto degree of overidentification is lower. Each IRF is well approximated by two moments: the jump on impact and the persistence.

[31] In practical terms, we impose a very large weight on the moment conditions for loan charge-off rates and bank failure rates (associated with σ and v_o), such that the estimation is forced to pick parameters to hit those moments exactly.

[32] See the FDIC website here.

[33] Note that $\rho \neq \rho^I$ implies that bankers' and investors' valuation of bank equity differs, capturing reduced-form agency frictions. To keep the paper concise, we have opted not to focus on the incentive issues with delayed accounting. Corbae and Levine (2018) are the first to provide a quantitative assessment of regulatory policies modulated by agency frictions.

[34] Note that even though the estimated value of ρ^I is higher than ρ, our model is still consistent with agency models such as the model in Acharya and Thakor (2016), where the agent (banker) has a higher effective discount rate than the principal (investor). This is because the banker's objective includes curvature, increasing the effective discount rate of the banker above ρ^I. A useful benchmark is to consider the value of s under immediate accounting and $\theta = 1$. In that case, $Q = s = \frac{\rho}{\rho^I - \Omega^W}$. Thus, the estimated value of ρ^I is influenced by the dividend rate and the growth rate of equity as well as the data target for Q. For the target value of Q and the growth rate of equity induced by the joint estimation, it is easy to verify that $\rho^I > \rho$. Given our estimation, the banker's effective discount rate is $\rho + \theta \cdot \Omega^W = 0.0224 + 7.94 \cdot 0.02 = 18.12\% >> 3.47\% = \rho^I$.

[35] Since we also target the IRF for liabilities and log market leverage is defined as the difference between log liabilities and log market equity, this is equivalent to targeting the IRF of market equity.

we estimate to be 4.16%. The interpretation is that approximately 65% of unrecognized losses are recognized within 10 quarters. It is reassuring that the delay estimated from the cross-section is consistent with the time series since net charge-offs taper off by the end of 2010, approximately two-and-a-half years after the trough in market values.

Finally, the value of κ determines the number of banks for which market-based liquidation is a concern. Banks located in the flat region of the shadow boundary (those with a high z) exhibit an immediate response in their liabilities to a loan default shock. Therefore, the initial jump in the IRF of liabilities provides insight on the proportion of banks on the flat region of the shadow boundary and, by extension, on κ. Appendix F shows that the model fits the data well.

4.2 Matching Facts

We evaluate the model's ability to reproduce the four facts from Section 2, focusing on the period between 2007 Q3 and 2019 Q4, during which banks experienced a large credit shock followed by a slow recovery.

Aggregate Shocks. We add aggregate shocks to our model to match the time series of Facts 1 and 3.[36] To back out a shock time series that mimics the global financial crisis (GFC), we subject the values of three parameters, σ, α, and v_o, to an unanticipated shock, starting the model from the stationary distribution. That is, we choose the shocked values of these parameters such that the model approximately matches the aggregate net charge-off rates of bank loans—Panel (a) of Figure 7—and the cumulative bank failure rate by 2019 Q4 of 7.52%.[37] Banks learn in 2007 Q3 that the values of σ, α and v_o will be different for 10 (σ) and 50 (α and v_o) quarters, respectively, including 2007 Q3. Afterward, all three parameter values revert back to their baseline values in Table 1. Specifically, we first assume that the arrival rate of loan default shocks σ jumps from the estimated value of 0.115 to $\sigma^{GFC} = 0.805$ between 2007 Q3 and 2009 Q4. After 2009 Q4, the arrival rate jumps back to $\sigma = 0.115$. Second, we assume that the speed of loss recognition jumps from the estimated value of $\alpha = 4.16\%$ to $\alpha^{GFC} = 9.78\%$ in 2007 Q3 and remains at this level until 2019 Q4, after which it reverts to α.[38] This captures the increased regulatory scrutiny of banks during and after the GFC, which forced banks to recognize losses more quickly. Finally, we also change the value of the bankers' outside option, the liquidation value, from its calibrated

[36] Note that with idiosyncratic shocks and a continuum of banks, the law of large numbers guarantees that the aggregate time series generated by the model are deterministic.

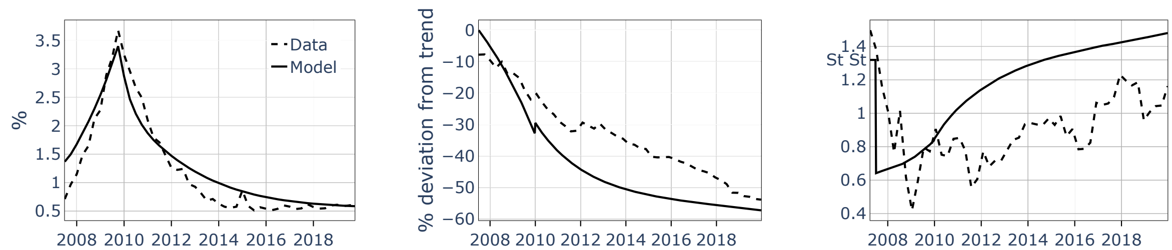
[37] Based on FDIC data, 548 banks failed between 2007 and 2019, almost all of them before 2012, and there were 7288 banks in 2007. Thus, we target a cumulative bank failure rate of 548/7288=7.52%.

[38] The increased value of α until 2019 Q4 allows the model to capture the decline in the net charge-off rates post-2010.

value of $v_o = 0.046$ to a value over the period 2007 Q3 to 2019 Q4 of $v_o^{GFC} = 0.01$. [39] Panel

Figure 7: Crisis and Recovery in Model and Data

(a) Net Charge-off Rate (b) Loan Book Values (c) Tobin's Q (Fact 1)



Notes: This figure compares the aggregate series of model-generated data (solid line) to the empirical data (dashed line). These series are based on a simulation that feeds in shocks chosen to match the times series of aggregate net charge-off rates (Panel (a)). Panel (b) presents the evolution of book loans as a deviation from a trend based on the 10 years before 2006 Q4. Panel (c) presents Tobin's Q. We plot the steady-state value of Tobin's Q for 2007 Q2—before the aggregate shock is realized—as a reference. The labels on the x-axis refer to the first quarter of the year.

(a) of Figure 7 shows that these parameter assumptions generate a good fit of the aggregate net charge-off rate series. We also closely hit the cumulative bank failure rate at 7.58% relative to 7.52% in the data. In addition, the model reproduces the untargeted decline in the book value of loans, as Panel (b) of Figure 7 shows.[40] Our GFC shock causes aggregate loan book values to shrink by approximately 50% relative to trend, which is similar to the change in the data.

We use these values to show how the model fits Facts 1 and 3 from Section 2.

Fact 1. Market and Book Equity Value Divergence. Fact 1 is that the aggregate market value of bank equity differs from aggregate book equity, with particularly divergent dynamics during crises. Panel (c) of Figure 7 plots the time series of Tobin's Q, the ratio of market equity to book equity, in the data (dashed line) and compares it to that in the model with aggregate shocks (solid line). The delayed loan loss recognition mechanism generates a sustained and pronounced decline in Tobin's Q of more than 50% on impact. This is driven predominantly by little q, the ratio of fundamental equity to accounting equity; we do not assume changes in investors' discount rate ρ^I. In the data, Tobin's Q falls more gradually by more than 70%, bottoming out at the end of 2008. In the model, banks learn the path of aggregate shocks in the third quarter of 2007. As a result, the response of Q is concentrated at the very beginning.

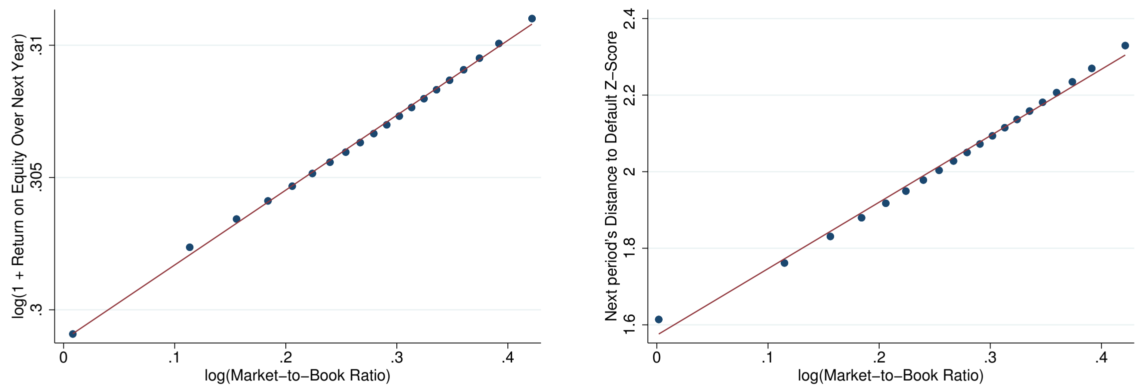
[39] The reason we need a lower v_o is that a higher loan default arrival rate and higher loan loss recognition rate would result in too many bank failures. Lowering v_o reduces the attractiveness of bankruptcy.

[40] Because book loans are growing, we detrend book loans in the model using the steady-state growth rate and the data using the exponential trend of the 10 years prior to 2007 Q3.

Fact 2. Predictive Power of Tobin's Q. The second stylized fact is that banks' Tobin's Q predicts their book ROE and the D2D measure, indicating that market values contain information about future cash flows that books do not. The model captures this predictability because market values contain information on unrecognized losses embedded in fundamental values. In Panels (a) and (b) in Figure 8, we show that the model generates the same upward-sloping relation between Tobin's Q and future ROE and D2D observed in the data.

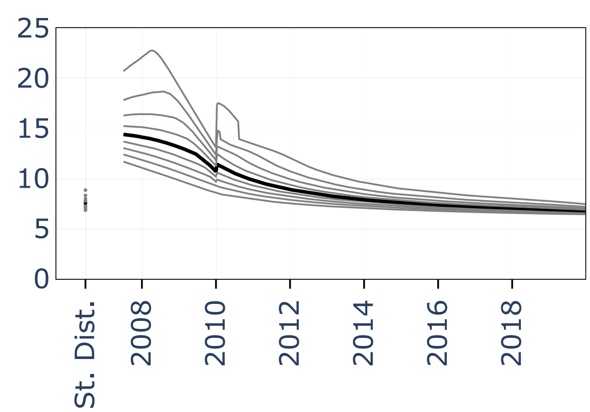
Figure 8: Predictability (Fact 2)

(a) Return on Equity (b) Distance to Default



Notes: Panel (a) presents a cross-sectional binscatter plot of next-year's ROE over log Tobin's Q (log market-to-book ratio of equity). Panel (b) presents a cross-sectional binscatter plot of the distance to default over log Tobin's Q (log market-to-book ratio of equity). Both figures control for the book value of equity and equity capitalization. The data are from model-simulated data using the stationary distribution and the parametrization in Table 1.

Figure 9: Market Leverage Dispersion (Fact 3)



Notes: This figure shows the distribution of market leverage for model-simulated data in response to the same aggregate shocks as in Figure 7. The bold line is the median, and the thin lines are cross-sectional deciles of market leverage. The stationary distribution of market leverage is plotted for reference. The year labels on the x-axis refer to the first quarter of the year.

Fact 3. Constraints. The third stylized fact is that banks avoid hitting the regulatory constraint and the market constraint by keeping a book equity buffer over the regulatory limit and are far from the market leverage constraint, which allows for an increase in the cross-sectional dispersion in market leverage during crises. We purposefully designed the model to capture the capital buffer over the regulatory minimum—recall Figure 6. Feeding in the aggregate shocks to the three parameters as discussed above, Figure 9 shows that we can also capture the increase in the cross-sectional dispersion of market leverage during the GFC, though not its full extent. Note that once the default arrival rate σ returns to its estimated value of 0.115, banks take on more risk by levering up. Our model abstracts from many features in the data that would induce more cross-sectional dispersion in market leverage, such as ex ante heterogeneity, fat-tailed default shocks, and time-varying investor risk premia. Nevertheless, our model captures approximately one-third of the increase in the leverage dispersion and the prolonged effects of the GFC.

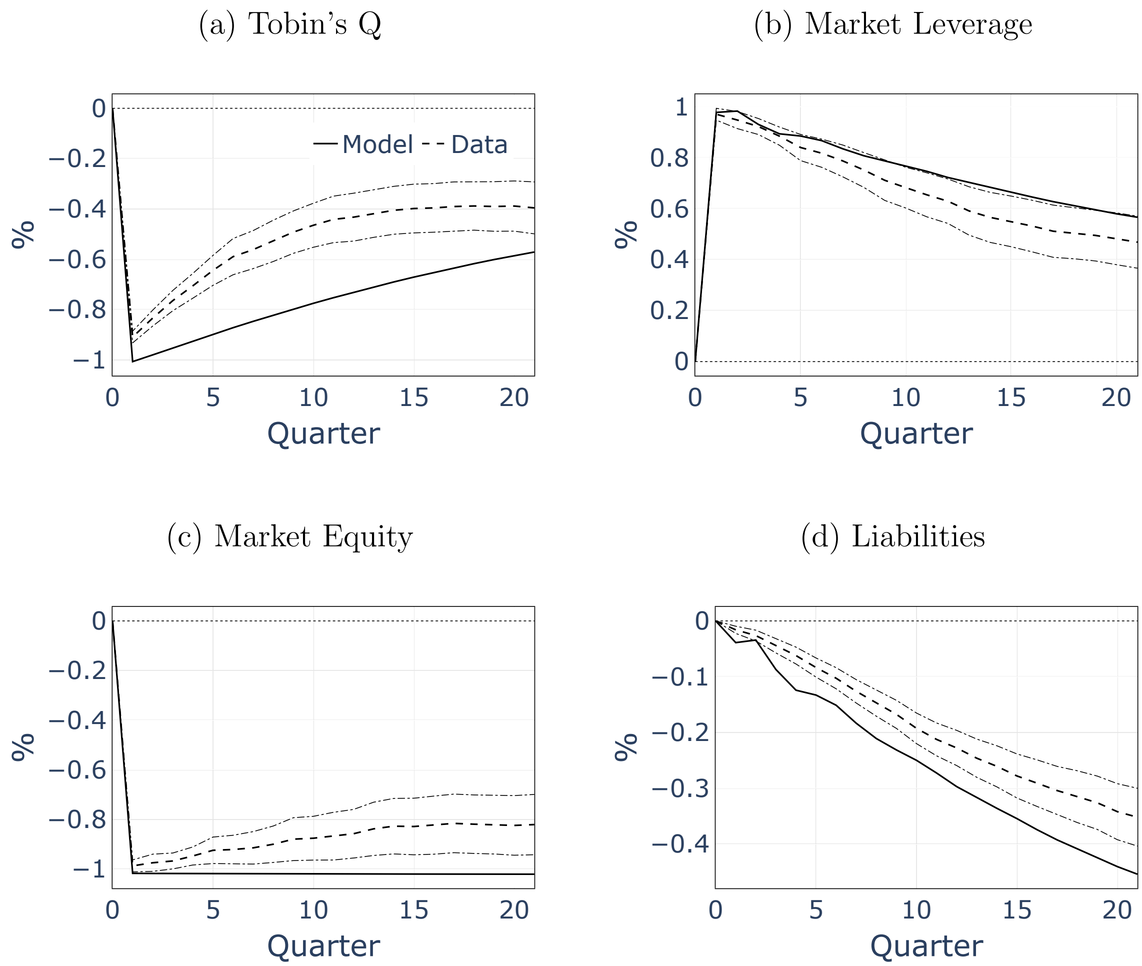
Fact 4. Slow Leverage and Tobin's Q Dynamics. Figure 10 compares the IRFs of the data (dashed lines) with those generated by the model (solid lines).[41] We show the IRFs of Tobin's Q in Panel (a), market leverage in Panel (b), market equity in Panel (c), and total liabilities in Panel (d). All plots also include the 95% confidence bands on the data IRFs. Our model reproduces the slow return to pre-shock levels in Tobin's Q via the dynamics of little q, whereby the defaults are only slowly recognized in accounting values relative to fundamental values—see Panel (a). We can also generate an IRF of market leverage that is close to the data—see Panel (b). Panel (c) shows that the model reproduces the slow recapitalization process following a negative shock: market equity does not recover at all in the model, while it recovers by only 20% in the data after five years. Finally, Panel (d) shows that our model captures the slow decline in banks' liabilities in the data.

4.3 Effects of Accounting Rules

The Current Expected Credit Loss (CECL), a new accounting standard, went into effect for all financial institutions in 2023. Most publicly traded banks have had to adhere to CECL accounting since January 2020, though the Coranavirus pandemic gave those banks an option to delay. CECL requires banks to estimate and record expected credit losses over the life of a loan at the time of origination or acquisition. This forward-looking approach contrasts sharply with the previous incurred loss model, which recognized losses only after they became probable. As a result, CECL encourages earlier recognition of credit losses and promotes greater transparency in financial reporting. In this section, we show that

[41] To compute the model IRFs, we first solve and simulate the model using the baseline parameter values from Table 1 and construct bank market returns as explained in Section G.2 Eq. (72). We run pooled ordinary least squares (OLS) regressions of the demeaned variable of interest on banks' market return and 20 lags using the simulated data. Finally, we take the coefficients, multiply each one by -1%, and compute the IRF at horizon h as the sum of the coefficients up to lag h.

Figure 10: Model vs. Data Impulse Responses (Fact 4)

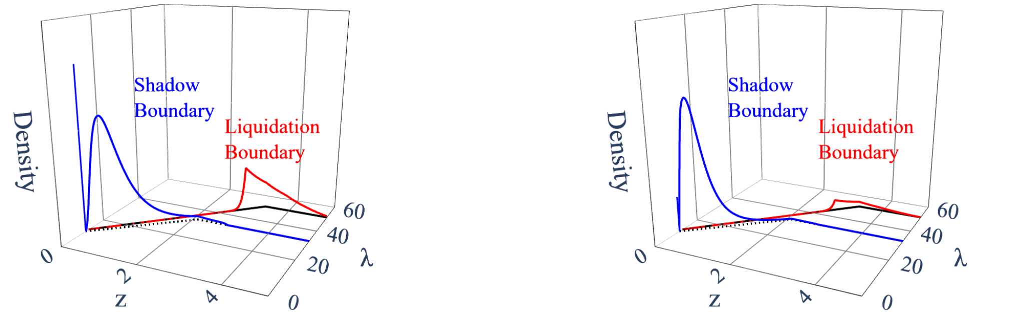


Notes: The figures present the impulse response functions of model-simulated data (solid line) for the benchmark calibration and compares them to those from the data (the dashed line represents the point estimates and the dash-dot lines the 95% confidence interval). We show the impulse response function of Tobin's Q in Panel (a), market leverage in Panel (b), market equity in Panel (c), and liabilities in Panel (d). To compute the model IRFs, we first solve and simulate the model using the baseline parameter values from Table 1 and construct bank market returns as explained in Section G.2 Eq. (72). We run pooled OLS regressions of the demeaned variable of interest on banks' market return and 20 lags using the simulated data. Finally, we take the coefficients, multiply each by -1%, and compute the IRF at horizon h as the sum of the coefficients up to lag h.

an accounting reform in the spirit of CECL, i.e., which accelerates the speed of loan loss recognition, features a tradeoff.

Figure 11: Comparison of Stationary Distributions for Different α

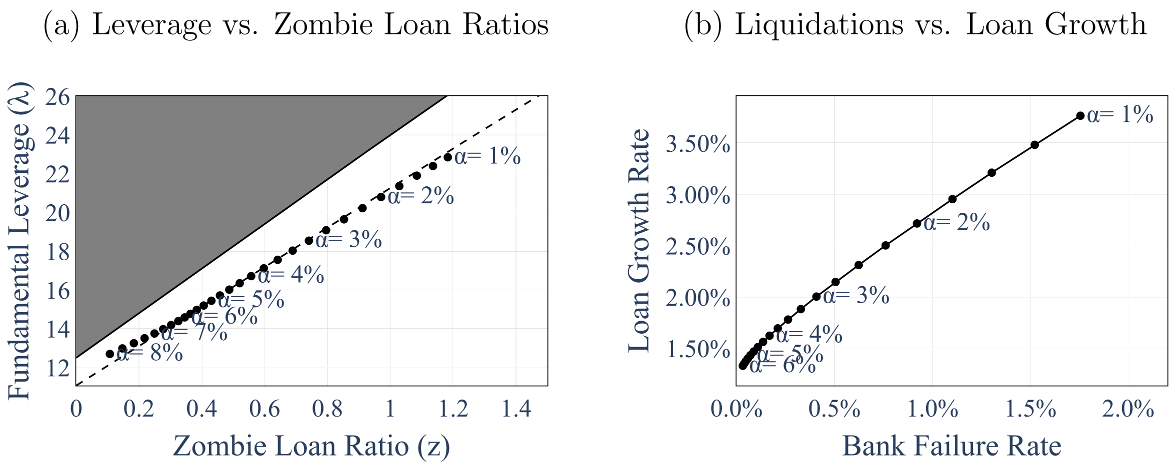
(a) Baseline Parameters: $\alpha = 4.16\%$ (b) Faster Recognition: $\alpha = 6\%$



Notes: This figure presents a two-dimensional plot of the stationary distribution of banks across fundamental leverage λ and the zombie loan ratio z. The black dashed line traces out the shadow boundary $\Lambda(z)$ and the solid black line the liquidation boundary $\Gamma(z)$. The blue and red lines are the density of banks conditional on their choosing the shadow and liquidation boundaries, respectively. For visualization purposes, the density conditional on banks' choosing the liquidation boundary has been multiplied by 20, as it is otherwise not visible. Panel (a) sets parameters at their benchmark levels in Table 1, and Panel (b) increases α to 6%.

We capture an accounting rule policy change through changes in α. A relaxation of accounting standards increases loan growth in our model while also increasing bank liquidation risk—and vice versa for a tightening of accounting rules in the spirit of CECL. To understand why this is the case, recall that the zombie loan ratio z drifts toward zero at rate $\mu^z = -z\left(\alpha + \mu^W\right)$ and that equity growth μ^W depends on the levered return. In turn, the jump in z increases with leverage. Thus, for any initial value of z_0, the expected value of z_t should lower with α, conditional on the bank's surviving. However, higher values of z lead to more frequent liquidations, shifting the mass of banks toward $z = 0$, as failed banks are replaced with new banks initialized at $z = 0$. To visualize these distributional changes, Figure 11 plots the density of $\{z, \lambda\}$ for two values of α. The density is plotted on the z-axis, whereas the y- and x-axes represent the leverage and zombie loan ratios, respectively. The shadow boundary (in blue) and liquidation boundary (in red) are projected onto the x–y plane of Figure 6. The invariant distribution of banks resides on the liquidation and shadow boundaries. When we compare Panel (a) with Panel (b), an increase in the loss recognition rate α translates into a greater mass of banks with lower fundamental leverage and, consequently, lower equity and loan growth. On the flip side, lower values of z also decrease liquidations, as fewer banks are on the liquidation boundary. This is the source of the tradeoff between loan growth and bank liquidation risk that we discuss next.

Figure 12: Steady States for Different α: Effect of Delayed Loss Recognition on λ and z



Notes: Panel (a) presents cross-sectional averages of λ and z from the stationary distribution for different values of α. The gray shaded area presents the liquidation set and the dashed line the shadow boundary. Panel (b) presents the cross-sectional average of banks' loan growth rate and the average bank failure rate for different levels of α. We assume that all other parameters remain at their benchmark levels shown in Table 1.

To illustrate this tradeoff, Panel (a) of Figure 12 presents the steady-state cross-sectional averages of the zombie ratio z and fundamental leverage λ obtained from the stationary distribution of banks for two values of α. There is a clear negative relation between α and the average levels of z and λ. Strikingly, although the fundamental leverage ratio λ differs for different α, the average book leverage is essentially identical in each case: book leverage ranges from 11.05 with $\alpha = 1\%$ to 11.59 with $\alpha = 8\%$. This occurs because most banks remain at the shadow boundary of the regulatory constraint. Hence, all of these economies look similar in terms of accounting values, while the fundamental leverage and liquidation risk differ significantly.

Panel (b) of Figure 12 shows how changes in α induce a policy tradeoff between liquidation risk and loan growth. The graph shows that there is a range of values $\alpha \leq 6\%$ for which faster loan loss recognition induces a decline in growth of lending and bank failures. A policy tradeoff is present since banks do not internalize the social costs of liquidation when taking risk. Having clarified this tradeoff, we move to the normative implications of our model.

5 Policy Implications

In this section, we investigate the normative implications of changes in accounting standards. We introduce an appropriate welfare notion to the theoretical model and then use our estimation to derive normative implications quantitatively.

5.1 Normative Analysis

We embed our bank Q-theory into general equilibrium, resulting in a microfounded social welfare function.

Nonfinancial Agents. We provide a full description of the nonfinancial sector and the derivations of the social welfare function in Appendix C. Here, we summarize the environment. A representative risk-neutral household holds wealth in bank stocks and capital in a production sector. In the spirit of He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014), banks specialize in loans, an essential source of funding for a most productive sector that households cannot directly fund. Households can fund a less productive sector.

Capital in the loan-funded sector is randomly destroyed, leading to the loan defaults encountered earlier. When bank equity is scarce, which we assume, the return to capital for the less and most productive sectors generates perfectly elastic deposit supply and loan demand curves, like those in Section 3.

The key assumption motivating regulation is that banks do not internalize the social costs of liquidations. When a bank is liquidated, loan losses, which are only ε if the bank survives, increase to $\varepsilon + (1-\psi)(1-\varepsilon)$ if the bank is liquidated—$\psi < 1$ captures bank restructuring costs.[42] We assume the social cost of liquidation is large enough that risking liquidation is never socially desirable:

Assumption 2 *Risking liquidation is socially inefficient:* $r^L - r^D \leq \sigma(\varepsilon + (1-\psi)(1-\varepsilon))$.

Social Welfare. A social welfare function aimed at maximizing the representative household's welfare can be simplified to maximizing the present value of aggregate bank dividends:

$$\mathcal{P}(\alpha, \Xi, \{g_0\}) \equiv \int_0^\infty \int_0^\infty \mathbb{E}\left[\int_0^\infty \exp(-\rho t)\, cW_t dt \middle| W_0 = W, z_0 = z\right] g_0(z, W)\, dz dW, \quad (16)$$

where $g_0(z, W)$ is the initial joint distribution of z and W. The expectation considers the formation of new banks after banks are liquidated. In contrast to the bank's private objective, the planner internalizes the social costs.

Immediate Accounting – Normative Analysis. To develop intuition, we solve for the optimal capital requirement under immediate accounting, distinguishing between the socially optimal leverage and the optimal capital requirement. It turns out that under immediate

[42]Namely, when a bank is liquidated, the social losses are not only ε but also the additional loss $(1-\psi)$ on the remainder of the bank's loans. Bankruptcy spillovers are discussed in Bernstein, Colonnelli, Giroud and Iverson (2019).

accounting, the objective in 16 can be written as a static risk-return tradeoff that dictates the social return on leverage. Because of scale independence, the solution pins down the same optimal leverage across banks. We distinguish between first-best leverage, i.e., that chosen by the planner, and the second-best regulation, which does not directly control leverage but anticipates the banks' best response to the regulation.

Proposition 3 *[Optimal Regulation] The first-best leverage and second-best regulation are given by the solution to the following optimization problems:*

1. **First Best: Socially Optimal Leverage.** *Let the optimal (first-best) leverage λ^{fb} be the socially optimal leverage λ considering only market-based liquidations. The first-best leverage solves:*

$$\Pi^{fb} = \max_{\lambda} \underbrace{\left(r^L - r^D - \sigma\left[\varepsilon + (1-\psi)(1-\varepsilon)\,\mathbb{I}_{[\lambda>\Lambda]}\right]\right)}_{\textit{social return of leverage}} \lambda. \tag{17}$$

The optimal leverage is $\lambda^{fb} = \kappa\left(1+\varepsilon(\kappa-1)\right)^{-1}$.

2. **Optimal Capital Requirements.** *Let the optimal (second-best) capital requirement Ξ^* be the socially optimal value of Ξ, taking as given the bank's optimal response (13) and considering both regulatory and market-based liquidations. The optimal capital requirement solves:*

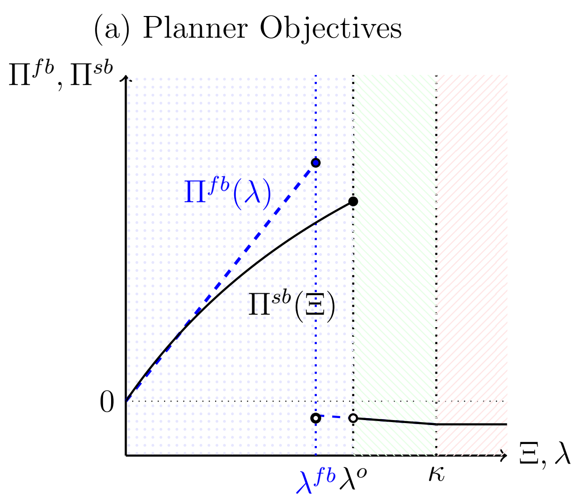
$$\Pi^{sb} = \max_{\Xi} \underbrace{\left(r^L - r^D - \sigma\left[\varepsilon + (1-\psi)(1-\varepsilon)\,\mathbb{I}_{[\lambda^*(\Xi,\kappa)>\Lambda]}\right]\right)}_{\textit{social return of leverage}} \lambda^*(\Xi,\kappa). \tag{18}$$

2.a. First Best: Laissez-Faire Regulation. *If under laissez faire banks do not risk liquidation, $\kappa \le \lambda^o$, then laissez faire achieves the first best.*

2.b. Second Best: Capital Requirements. *If under laissez faire banks risk liquidation, $\kappa > \lambda^o$, then the first-best solution is unattainable, and $\Pi^{sb} < \Pi^{fb}$. The optimal capital requirement is $\Xi^* = \lambda^o$, and banks set leverage at the shadow boundary, $\lambda^{sb} = \lambda^o\left(1+\varepsilon(\lambda^o-1)\right)^{-1}$. Thus, if regulation is warranted, leverage is lower than the first best, $\lambda^{sb} < \lambda^{fb}$.*

The proposition clarifies the role of capital requirements. Under immediate accounting, the socially optimal leverage is given by a static risk–return tradeoff, encoded in the social return of leverage. Because it is socially desirable to avoid liquidations, the planner sets first-best leverage at the shadow boundary of the market-based constraint: the value that maximizes loan growth while avoiding liquidations. Recall from Section 3.2 that λ^o is the level of leverage at which banks switch from no risk-taking to risk-taking. When the market-based constraint is sufficiently tight, $\kappa \le \lambda^o$, banks set leverage at the shadow boundary, so laissez faire achieves the first best. When banks risk liquidations absent regulation—i.e., when $\kappa > \lambda^o$—capital requirements are warranted but cannot implement the first best.

Figure 13: First-Best Leverage and Optimal Regulation (Immediate Accounting)



Notes: These panels show the regulator's problem under immediate accounting, in terms of the planner's objectives (left panel) and the bank's behavior λ^* in response to regulation (right panel) for the case where $\kappa > \lambda^o$. The planner's objective under the first best is denoted by Π^{fb} and under the second best by Π^{sb}. The right panel shows the bank's best response.

Capital requirements cannot achieve the first-best because regulatory enforcement mandates socially inefficient liquidation. Figure 13 shows this in detail. In Panel (a), the dashed curve represents the first-best objective function, Π^{fb}, as a function of leverage, λ. This curve increases up to the first-best leverage, λ^{fb}, and turns negative beyond that point. The negative values arise because risking liquidations is socially suboptimal. As a result, λ^{fb} is the shadow boundary of the market-based leverage constraint (κ), the maximum leverage that avoids the risk of default.

The solid curve in Panel (a) depicts the planner's objective function under the second best, $\Pi^{sb}(\Xi)$, which is plotted as a function of the capital requirement, Ξ, instead of λ. The second-best objective has Ξ as an argument because regulation does not control leverage directly but instead influences it through the banks' best response to the capital requirement. Panel (b) shows the bank's best-response leverage $\lambda^*(\Xi)$: Banks set their leverage at the shadow boundary when $\Xi \leq \lambda^o$, and at the liquidation boundary when $\Xi > \lambda^o$.

The graph of Π^{sb} in Panel (a) is determined by the best response $\lambda^*(\Xi)$. If the regulator sets $\Xi > \lambda^o$, banks risk liquidation, leading to a socially inefficient outcome and a negative value for Π^{sb}. The regulator must ensure $\Xi \leq \lambda^o$ to prevent such liquidations. However, when $\Xi \leq \lambda^o$, banks keep a capital buffer and set their leverage at the shadow boundary of Ξ. This buffer prevents the implementation of first-best leverage. Consequently, the second-best outcome satisfies $\Pi^{sb}(\lambda^o) < \Pi^{fb}(\lambda^{fb})$ in the region without liquidation.

The optimal capital requirement, Ξ, is therefore set to λ^o, inducing banks to set leverage at $\Lambda(\lambda^o)$, which is the shadow boundary corresponding to λ^o. This choice maximizes second-best leverage while avoiding liquidation risks.

Intuitively, capital requirements are second-best instruments because they rely on regulatory enforcement that mandates liquidation if banks breach the imposed limit. This enforcement mechanism compels banks to hold excess capital as a buffer, reducing lending compared to the first best. Despite this inefficiency, capital requirements are superior to a laissez-faire approach. Next, we show that, beyond providing a good description of the dynamics of Tobin's Q and leverage, delayed accounting is a valuable additional regulatory tool.

Adjustment Speed and Optimal α. Because capital requirements are imperfect instruments, regulation may improve upon the second-best outcome under immediate accounting by exploiting α as a policy tool. Recall from Section 4.3 that α induces a tradeoff between loan growth and bank liquidation rates. As α approaches infinity, leverage will be set at the shadow boundary of Ξ, and there will be no liquidations. Under finite values of α, the capital buffer that inefficiently limits lending is relaxed, but bank liquidation risk is increased.

Solving analytically for the socially optimal $\{\alpha, \Xi\}$-mix requires solving for the intractable joint dynamics $\{z, W\}$. However, the social welfare function has a convenient HJB representation.

Proposition 4 *[Optimal Regulation] Let g_0 be the initial joint distribution of $\{z, W\}$. The regulation with delayed accounting maximizes*

$$\mathcal{P}^*(\{g_0\}) \equiv \max_{\{\alpha,\Xi\}} \mathcal{P}(\alpha, \Xi, \{g_0\}) = \max_{\{\alpha,\Xi\}} \int_0^\infty W \int_0^\infty p(z)\, g_0(z, W)\, dz dW, \tag{19}$$

where $p(z)$ is the social value of a bank, which satisfies:

$$\rho p(z) = c + p_z(z)\, \mu^z + p(z)\, \mu^W + \sigma J^p(z), \quad \textit{and}$$

$$J^p(z) = \left[p(z + J^z)(1 - \varepsilon\lambda)\, \mathbb{I}_{[\lambda \leq \Lambda(z)]} + p(0)(1 - (\varepsilon + (1 - \psi)(1 - \varepsilon))\lambda)\, \mathbb{I}_{[\lambda > \Lambda(z)]} - p(z)\right].$$

The socially optimal $\{\alpha, \Xi\}$-mix maximizes the g_0-weighted average of the social value of an individual bank, $p(z)$. The function $p(z)$ is the present value of bank payouts. Notice that the social value is isomorphic to the private value, except that the planner internalizes the liquidation cost in the jump term. This representation allows us to obtain the optimal policy numerically.

5.2 Microprudential Implications: Optimal Regulation

In this subsection, we study the microprudential policy implications of our model and derive the optimal combination of $\{\alpha, \Xi\}$ numerically using our estimated model. This also allows for a normative assessment of speedier loss recognition rules, as implied by the recent move to the CECL accounting model. To this end, we consider the objective of maximizing social welfare after a one-time change in both α and Ξ, starting from the estimated stationary distribution and transitioning to a new stationary distribution after the policy change. We discuss the results in terms of Ξ^{-1}, which translate into capital requirements.

Figure 14: Optimal Microprudential Policy and Isovalues

Notes: Panel (a) shows the values of social welfare $\mathcal{P}(\alpha, \Xi, \{g_0\})$ for each choice of Ξ and α. Panels (b) and (c) show the bank failure rates and the cross-sectional average of banks' loan growth rates, respectively, at the stationary distributions of the different (Ξ, α) combinations.

From Section 5.1, we learned that the optimal policy maximizes the weighted average social value of banks, which includes the social cost of bank failures. Of course, it is not trivial to estimate the social cost of bank failures empirically. However, we can obtain an estimate by assuming that the status quo regulation has optimally set Ξ, given our estimated accounting rules. The implied social cost of banking failure then justifies the existing capital requirements under the current accounting rules as the optimal requirement. We find that the social cost supporting the rationale for current regulations is equivalent to an annualized negative dividend rate of -2.65%, which we hold constant across new combinations of $\{\alpha, \Xi\}$.[43]

Figure 14 summarizes the results. It presents contour plots for welfare in Panel (a), bank failure rates in Panel (b), and loan growth rates in Panel (c) as a function of α and Ξ. The two dots in each figure mark the estimated values and the socially optimal values. Recall from Figure 12 that, for a fixed Ξ, α governs a tradeoff between the frequency of bank liquidations and loan growth. At the optimal values of α and Ξ and relative to their estimated levels, lending growth rates are reduced by 46 basis points, while bank liquidation rates also decline by approximately 10 basis points. The optimal policy suggests that expediting loss recognition should be a regulatory priority: α is 17% at the optimum, compared to the baseline estimated value of 4.16%. To put these numbers in perspective, the reform would bring the half-life of zombie loans from four years to just a year and a half. However, the optimal policy couples this change with a looser capital requirement: the optimal capital requirement goes from the 8% mandated by Basel III standards down to approximately 5.5%, closer to the requirement under Basel II.

To understand what drives these welfare gains, we note that social welfare $\mathcal{P}(\alpha, \Xi, \{g_0\})$ is well approximated by the following aggregate bank moment[44]:

$$\mathcal{P}(\alpha, \Xi, \{g_0\}) \approx \frac{\mathcal{C} - \mathcal{S}}{\rho - (\mathcal{G} - \mathcal{C})}.$$

In this approximation, $\mathcal{C}$ stands for the aggregate dividend rate, $\mathcal{S}$ for the flow of social losses, and $\mathcal{G}$ for the aggregate ROE before dividends. The flow of social losses $\mathcal{S}$, which acts as a negative dividend, is approximately the failure rate multiplied by the present value of the social cost of liquidations, e.g., $\mathcal{S} \approx 2.65\%$. In the data, the failure rate of banks is very small. Hence, to justify the current level of capital requirements, the social cost of default must be large. As a result, welfare is sensitive to the failure rates even though the rates are low. When α is fixed at its estimated value of $\alpha = 4.16\%$, regulation can limit the flow of social costs only with tighter capital requirements. When regulators are given the additional tool of speeding up loss recognition by choosing an optimally higher value of α, they shift the distribution of z toward lower values and away from the liquidation boundary—recall

[43] The social cost of default is the jump term after a default event: $p(0)(1 - (\varepsilon + (1-\psi)(1-\varepsilon))\lambda(z))\mathbb{I}_{[\lambda > \Lambda(z)]}$ from the definition of $J^p(z)$ in Eq. (19). In our numerical exercises, most banks choosing $\lambda > \Lambda(z)$ choose the market-based liquidation boundary, and hence, $\lambda(z) = \kappa$. Using our estimated values, we obtain a value for this term of -1.17. Bank liquidations average 0.05% per year. The annuity value of a social loss is $-1.17/\rho = -1.17/0.0224$. Multiplying the liquidation rate of 0.05% by the annuity value translates the cost into a flow cost of -2.65% per year.

[44] For example, at the optimal regulation, the approximation differs from the numerical value by less than 1%.

the distribution shifts in Figure 11. Thus, with fewer zombie loans, capital requirements can be even relaxed without increasing liquidations. In contrast, an increase in α reduces bank liquidations by two-thirds, bringing $\mathcal{S}$ down from 2.65% to 1.8%. While banks would appear more levered under the reform—book leverage would increase from 11.1 to 15.1—this change is only cosmetic, as fundamental leverage increases only slightly from 16.5 to 17.1. Relaxing the bank capital requirements increases banks' ROE, $\mathcal{G}$, by approximately 1%. An increase in bank profitability without an increase in liquidations is possible because the reform reduces banks' incentives for hidden risk-taking and therefore narrows the cross-sectional distribution of leverage. It results in safer banks with book values closer to fundamental values. Safe banks reduce their excessively large capital buffers, while risky banks are forced to delever faster. The welfare gains from the reform are only somewhat mitigated by an undesirable increase in dividends, $\mathcal{C}$, which offsets the increase in $\mathcal{G}$.[45]

In sum, moving accounting values closer to fundamental values makes the banking system safer. In addition, our exercise suggests that accounting standards and capital regulation should be jointly optimized: tighter accounting standards require looser book regulations to target the same fundamental leverage. The next section explores the macroprudential implications of our model.

5.3 Macroprudential Implications: CCyB

In this section, we analyze the effects of an aggregate shock under three different regulatory regimes: (i) a constant capital requirement and delayed accounting as estimated in Section 4, (ii) a countercyclical capital buffer (CCyB) in the presence of delayed accounting, and (iii) a countercyclical accounting rule.[46] Our findings indicate that delayed accounting can lead to unintended consequences when a CCyB rule is imposed. Under delayed accounting, a relaxation of regulatory limits on leverage during crisis times can increase the risk-taking of banks and lead to more bank failures in the long run.[47] By contrast, relaxing accounting rules during a loan default crisis leads to fewer bank failures in our model.

We study the effects of a CCyB rule in our model by first simulating a boom during which the capital requirement tightens and then a bust during which the capital requirement is relaxed. To simulate a boom in our model, we introduce a time-varying loan default arrival

[45] Recall that as $\mathcal{G}$ increases, banks pay more dividends. This is because wealth effects dominate substitution effects for the estimated value of θ.

[46] For work on the economic effects of CCyB rules see Benes and Kumhof (2015), Gambacorta and Karmakar (2018), Faria-e Castro (2021), Simon (2021).

[47] According to the BIS, "Basel III requires that the CCyB be activated and increased by authorities when they judge aggregate credit growth to be excessive and to be associated with a build-up of system-wide risk. The buffer would subsequently be drawn down in a downturn to help ensure that banks maintain the flow of credit in the economy."

rate that is half the estimated value of $\sigma = 0.115$ at the peak of our simulated boom:

$$\sigma_t = \sigma \left(1 - \frac{1}{2}\exp(-\eta(t-\tau)^2)\right). \tag{20}$$

We choose the scalar τ such that σ_t is one-half of our estimated $\sigma = 0.115$ at the boom's peak, six quarters after banks learned about the shock. The scalar η governs the persistence of the shock that we choose such that, in the absence of other shocks, σ_t returns to its estimated steady-state value in approximately five years.[48] Six quarters into the boom simulation, at the peak, we hit banks with a second shock (at $t = 0$) that captures suddenly deteriorating credit conditions. We model a crisis as following a boom, where the probability of loan default shocks σ_t doubles to $2 \times \sigma$ over six quarters. Notably, at the onset, banks do not know that a crisis will happen at the boom's peak. In the figure, the boom shock hits banks at $t = -6$, and the crisis shock hits banks at $t = 0$.

Figure 15 shows how aggregate lending (left column) and bank failure risk (right column) respond to an aggregate shock under three policy regimes during the credit crisis and its aftermath. In all exercises, we do not replace failed banks, motivated by the idea that there is little bank entry during a banking crisis. Note that we plot aggregate loans as percentage deviation to the boom's peak at $t = 0$. The solid lines of the graphs in the left column represent the percentage deviation of aggregate loans from steady-state trend growth. The graphs in the right column show the fraction of banks operating at the liquidation boundary. We distinguish between "high-shock" banks, those hit by an above-average number of loan default events (dashed lines), and "low shock" banks, those hit by a below-average number of loan default events (dotted lines).[49]

The top row presents the results for our baseline policy regime of a constant capital requirement Ξ and delayed accounting based on our estimation. The credit crisis leads to a slow decline in aggregate lending (left panel) because a larger mass of banks moves to the liquidation boundary (right panel), where failure is imminent. Since failed banks lose their equity capital, aggregate credit supply declines since it equals levered aggregate equity.

In the middle row of Figure 15, we analyze a CCyB regime, which we model as a time-varying Ξ_t:

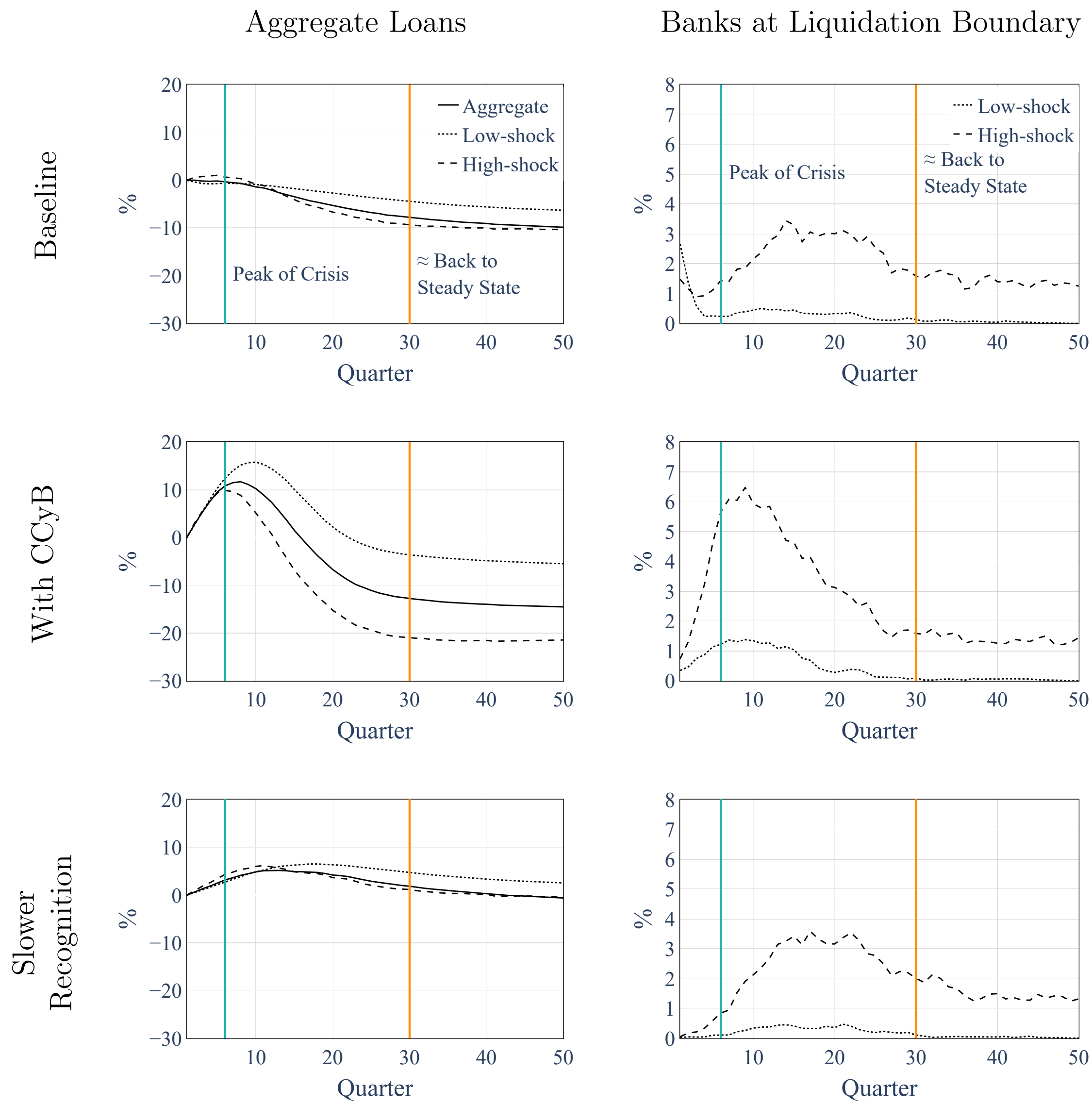
$$\Xi_t = \Xi \left(1 + \left(\frac{\Xi^{CCyB}}{\Xi} - 1\right)\exp(-\eta(t-\tau)^2)\right). \tag{21}$$

We choose the same values for η and τ as in Eq. (20), starting from the boom, where the capital constraint is progressively tightened from 8 to 10% during the boom period (quarters one through six), and then subsequently relaxed to 6% during the credit crisis.

[48] We start the simulation at the stationary distribution. Banks learn about the path of σ_t at $t = -6$.

[49] Recall that the default intensity σ_t operates i.i.d. across banks even though all banks' probability of being hit has increased.

Figure 15: Macroprudential Policy Effects



Notes: The figure depicts aggregate lending (left column) and the percentage share of banks at the liquidation boundary (right column) during a simulated credit crisis (as given by Eq. (20)) and its aftermath under three macroprudential policies. Aggregate lending is shown as a percentage deviation relative to the preceding boom's peak, normalized at $t = 0$, and detrended using the steady-state growth rate. The top row presents the baseline case, where Ξ and α are constant at their calibrated and estimated values, respectively. The middle row presents the case under *CCyB*, where the capital requirement, Ξ_t^{CCyB}, follows Eq. (21). The bottom row shows the outcome when the accounting rule α_t^{CCyB} is time-varying and relaxed after an adverse aggregate shock. The plot shows aggregate (fundamental) loans as solid lines, "low-shock" banks as dotted lines, and "high-shock" banks as dashed lines.

Ξ_t, thus, mirrors the path of σ_t.[50] Compared to the baseline scenario, the CCyB initially triggers a brief surge in lending but also more failures as the relaxation of the constraint incentivizes banks to increase leverage to boost returns. As the crisis progresses, aggregate lending decreases relative to the baseline scenario, settling at a lower trend. This results

[50] This is consistent with Basel III and current practice, where the CCyB varies between 0% and 2.5%.

from an increase in bank failures that deplete aggregate bank equity. Interestingly, the effect is not only driven by the high-shock banks. There are also more liquidations among the low-shock banks. When the CCyB eases, banks with a high zombie loan ratio z opt to risk liquidation by increasing leverage. In sum, CCyB increases risk-taking behavior across the board, resulting in an initial increase in lending that is eventually offset by a larger number of bank failures. Contrary to the intended effects of the CCyB, the policy amplifies financial fragility.

The bottom row of Figure 15 presents the results of a policy that slows down the recognition of loan losses. We study a time-varying α, denoted as α_t^{CCyB}, that takes the same shape as Ξ_t in Eq. (21). In this case, we relax α from the estimated value of 4.16% to 2.16% at the peak of the crisis. The policy induces a much smoother lending series than the CCyB regime. Although relaxing accounting rules during credit risk events also induces more risk-taking than the benchmark, risk-taking is much lower than in the CCyB regime. This is because the countercyclical accounting rule primarily targets high-shock banks. Low-shock banks have fewer zombie loans on average, so the accounting relaxation does not strongly incentivize them to take on risk. Lending declines by less than in the benchmark because easing up on loan loss recognitions postpones the deleveraging of high-shock banks. As a result, our findings suggest that relaxing loss recognition rules during periods with increased default risk can be a more targeted policy than the CCyB in this setting.

Although delayed accounting and the CCyB are similar in that they both relax the regulatory constraint in the event of a negative shock, they differ in a critical respect. The CCyB is not conditional on the bank receiving adverse shocks. The CCyB policy does not target the banks most needing a relaxation of capital rules or could relax the capital constraints for a bank just *before* it is hit with a shock, which encourages more risk-taking. In contrast, delayed accounting delivers regulatory relaxations only conditional on receiving a negative shock. This makes delayed accounting a more targeted and better timed policy, creating better incentives for banks. Under delayed accounting, the bank cannot choose to lever up ex-ante; instead, it can increase its fundamental leverage only ex-post, in the dire state of the world where it is hit with the shock.

6 Conclusion

This paper presents four facts about banks' Tobin's Q and leverage. Motivated by these facts, we propose a heterogeneous bank model that distinguishes accounting, fundamental, and market values of bank equity and subjects banks to market constraints and book-based regulatory constraints. The novel feature of our theory is that banks delay the recognition of losses on their books. Delayed accounting of losses in conjunction with the book and market constraints allows the model to reproduce the four facts.

Our model reveals several novel policy implications. A regulatory reform designed to accelerate loss recognition induces a tradeoff between financial fragility and growth. Also, we show that a countercyclical capital buffer can make the banking system more fragile under delayed loss accounting. Our model stresses the necessity of bridging the gap between regulatory reliance on book values and the market's focus on fundamental values to achieve a more comprehensive understanding of bank dynamics and support regulatory design.

A limitation is that maintaining zombie loans does not cost resources and keeps inefficient firms alive. We also do not consider exogenous fluctuations in market values: forcing banks to recognize losses on marketable securities may induce excessive volatility if prices have non-fundamental components. Another limitation is that banks are treated in isolation in our model: in practice, banks have interconnected risk exposures and are subject to fire-sale externalities. Finally, we do not allow banks to choose how they adhere to accounting standards. Incorporating these features may further open important lines of research.

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For Online Publication
Online Appendix
"A Q-Theory of Banks"

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A Data Appendix

A.1 Sample Selection

We analyze bank holding companies (BHCs). BHCs file FR-Y-9C forms if they have assets above one billion dollars. Prior to 2015 Q1, this threshold was $500 million and prior to 2006 Q1, this threshold was $100 million. We focus on the sample period from 1990 Q1 to 2021 Q1.

We focus on top-tier bank holding companies that are headquartered in the 50 states or in Washington D.C. For book variables, we use data from the FR Y-9C, downloaded through Wharton Research Data Services (WRDS). We match this to data on market capitalization and returns from the Center for Research in Securities Prices (CRSP) using the PERMCO-RSSD links data set provided by the New York Fed (https://www.newyorkfed.org/research/banking_research/datasets.html). For analyses that use solely book data, we use data for those BHCs that we find in our sample in the FR Y-9C; for analyses that use market data, we use only the observations which we observe in both FR Y-9C and CRSP. In one robustness check, we use information on the dates of, and participants in, bank mergers and acquistions; we obtain data on bank mergers from the Chicago Fed (https://www.chicagofed.org/banking/financial-institution-reports/merger-data). In an additional robustness check, we drop all banks that were ever stress-tested (CCAR and DFAST). We obtain information on whether banks were ever stress tested from the Federal Reserve (The main website is https://www.federalreserve.gov/supervisionreg/stress-tests-capital-planning.htm, and the specific data sets can be found at https://www.federalreserve.gov/supervisionreg/ccar.htm and https://www.federalreserve.gov/supervisionreg/dfa-stress-tests.htm).

A.2 Motivating Facts

A.2.1 Regulatory Rules

Under Basel II (the regulatory standard in place during the crisis), bank holding companies were subject to regulatory minimums on their total capital ratio and their tier-1 capital ratio. These capital ratios are computed as qualifying capital/risk-weighted assets and, thus, a bank with a higher capital ratio has lower leverage. Basel II required that banks hold a minimum tier-1 capital ratio of 4% and a minimum total capital ratio of 8%. In order to be categorized as "well-capitalized," banks had to meet minimum capital ratios that were two percentage points higher (6% and 10%, respectively). Being categorized as well-capitalized is desirable because banks that are not well-capitalized are subject to additional regulatory scrutiny (Federal Deposit Insurance Corporation, 2022). After the crisis, tighter capital requirements were phased in under Basel III. The minimum total capital ratio stayed at 8% throughout our sample period, but the tier-1 capital ratio rose to 4.5% in 2013, 5.5% in 2014, and finally settled at 6% starting in 2015. Also under Basel III, additional capital ratios (e.g., tier-1 leverage and common equity capital ratio) began being monitored (however these ratios are quite similar to the preexisting tier-1 and total capital ratios) and, starting in 2016, a "capital conservation buffer" and special requirements for systemically important financial institutions were introduced (Basel Committee on Banking Supervision, 2011). Kisin and Manela (2016) study whether banks violate different regulatory constraints and find that typically banks do not violate multiple regulatory constraints.

A.2.2 Bank Accounting Practices

The discrepancy between book and market equity reflects bank accounting practices. Banks can delay acknowledging losses on their books (e.g. Laux and Leuz 2010), because banks are not required to mark-to-market the majority of their assets. There are many incentives to delay book losses. In practice, a key metric for measuring success of a bank is the book return on equity (ROE).[51] Given that ROE is a measure of success, manager compensation is linked to book value performance. Moreover, shareholders and other stakeholders may base their valuations on information from book data. Finally, banks are required to meet capital standards based on book values.

The flexibility of accounting their accounts is studied extensively in the accounting literature (Bushman, 2016 and Acharya and Ryan, 2016 review the literature on this issue, Francis, Hanna and Vincent, 1996 studies the same issue for non-financial firms). In practice, banks can record securities on the books using two methodologies: either amortized historical cost (the security is worth what it cost the bank to buy it with appropriate amortization) or fair value accounting.[52] In addition to mis-pricing securities, another degree of freedom is the extent to which banks can acknowledge impairments: banks have the right to delay acknowledging impairments on assets held at historical cost, if they deem those impairments as temporary (i.e. they believe the asset will return to its previous price). This gives banks substantial leeway, and led banks to overvalue assets on the books during the crisis. Huizinga and Laeven (2012) find that banks used discretion to hold real-estate related assets at values higher than their market value. (Laux and Leuz, 2010) note some notable cases of inflated books during the crisis: Merrill Lynch sold $30.6 billion dollars of CDOs for 22 cents on the dollar while the book value was 65 percent higher than its sale price. Similarly, Lehman Brothers wrote down its portfolio of commercial MBS by only three percent, even when an index of commercial MBS was falling by ten percent in the first quarter of 2008. Laux and Leuz (2010) also document substantial underestimation of loan losses in comparison to external estimates.

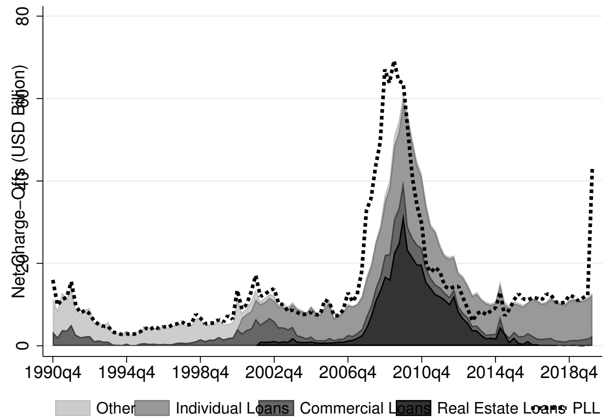
This shows up in our own analysis as well: Figure A.1 shows that provisions for loan losses and net charge-offs only reached their peak in 2009 and 2010 respectively, and remained quite elevated at least through 2011, well after the recession had ended. The decomposition of net charge-offs shows that these losses were heavily driven by real estate, suggesting they were associated with the housing crisis. Loan loss provisions lead net-charge-offs, which can be best seen for the 2008/2009 crisis and in the beginning for the Covid crisis (note we present data until 2020 Q1). Banks' books were only acknowledging in 2011 losses that the market had already predicted when the crisis hit. Harris, Khan and Nissim (2013) construct an index, based on information available in the given time period, that predicts future losses substantially better than the allowance for loan losses.[53] This implies that the allowance for loan losses is not capturing all of the available information to estimate losses. This may in part be strategic manipulation, but there may also be a required delay in acknowledging loan losses. Under the "incurred loss model" that was the regulatory standard during the crisis, banks are only allowed to provision for loan losses when a loss is "estimable and probable" (Harris et al., 2013). Thus, even if banks know that many of their loans will eventually suffer losses, they were not supposed to update their books until the loss was imminent.

[51] For example, JP Morgan's 2016 annual report states "the Firm will continue to establish internal ROE targets for its business segments, against which they will be measured" (on page 83 of the report).

[52] Fair value accounting can be done at three levels: Level 1 accounting uses quoted prices in active markets. Level 2 uses prices of similar assets as a benchmark to value assets that trade infrequently. Level 3 is based on models that do not involve market prices (e.g. a discounted cash flow model). Banks are required to use the lowest level possible for each asset. In practice, most assets are recorded at historical cost. The majority of fair value measurements are Level 2 (Goh, Li, Ng and Ow Yong 2015; Laux and Leuz 2010). Recent work has shown that the stock market values fair value assets less if they are measured using a higher level of fair value accounting. This leaves room to mis-price assets on books. Particularly during 2008, Level 2 and Level 3 measures of assets were valued substantially below one. Laux and Leuz (2010) document sizable reclassifications from Levels 1 and 2 to Level 3 during this period. They highlight the case of Citigroup, which moved $53 billion into Level 3 between the fourth quarter of 2007 and the first quarter of 2008 and reclassified $60 billion in securities as held-to-maturity which enabled Citi to use historical costs.

[53] The allowance for loan losses (ALL) is the stock variable corresponding to the provision for loan losses (PLL).

Figure A.1: Decomposition of Net Charge-offs



Notes: This figure shows aggregate net charge-offs for different categories (area chart) and aggregate loan loss provisions (dashed black line) from 1990 Q4 to 2020 Q1. The data source are FR Y-9C reports. Net charge-offs for loans are defined as charge-offs minus recoveries. We decompose the net charge-offs into loans backed by real estate, commercial and industrial (C&I) loans, loans to individuals (e.g., such as credit card loans), and all other loans (e.g. inter-bank loans, agricultural loans, and loans to foreign governments). Net charge-offs of loans to individuals are not separately recorded until 2001 Q1, and net charge-offs of real-estate loans are not separately recorded until 2002 Q1; these categories are thus grouped with "Other" until the date at which they are recorded separately.

A.2.3 Information content

There are at least two reasons to expect different information content in market and book value measures. One reason is the delayed acknowledgment of known losses, which is a widely documented fact in the accounting literature. As long as banks delay the recognition of losses, or refinance non-performing loans to avoid registering losses (evergreening), book values will not reflect banks' actual losses. If market participants can update their valuations more quickly, detecting these losses, differences in informational content will emerge. This alone can produce differences in the informational content of market and book equity. The other reason is that changes in the underlying market value of loans reflect default expectations while the book value of loans does not (see filing instructions for FR-Y-9C BHCs regulatory reports) at least until January 2020. Before 2020, loan loss expectations were not updated in loan accounting books and loans were only written off once the loss had occurred. Publicly traded banks were supposed to change their accounting system to the new "current expected credit loss" (CECL) accounting system in January 2020.[54] Note that our model can capture these accounting system changes (see Section 4.3) and study their effects on bank lending.

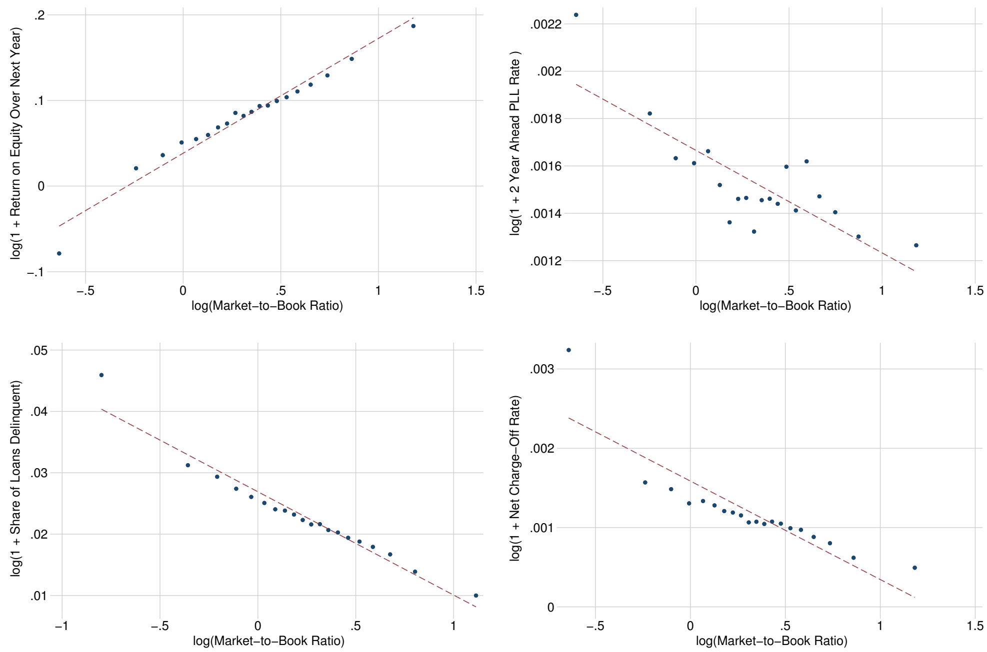
Figure A.1 suggests that indeed market values contain more information than book values of equity. Loan loss provisions, denoted as PLL in the figure, peak in early 2009 when market values had already tanked. Loan net charge-offs peaked even later in 2010 when the economy was no longer officially in a recession.[55] The decomposition of net charge-offs shows that these losses were

[54]See https://www.occ.treas.gov/news-issuances/bulletins/2021/bulletin-2021-20.html. However, on March 27, 2020, the Fed moved to provide an optional extension of the regulatory capital transition for the new credit loss accounting standard, see https://www.federalreserve.gov/newsevents/pressreleases/bcreg20200327a.htm.

[55]When a bank has a loss that is estimable and probable, it first provisions for loan losses on the income statement, which shows up as PLL in the figure. Later when the loss has realized, the asset is charged off

heavily driven by real estate, which is consistent with the nature of the crisis. Loan loss provisions lead net-charge-offs, which can be best seen for the 2008/2009 crisis and in the beginning for the Covid crisis (note we present data until 2020 Q1).

Figure A.2: Market equity contains more cash-flow relevant information than book equity



Notes: This figure presents cross-sectional binned scatter plots of log outcomes on the log Tobin's Q for BHCs. All plots control for log book equity as a proxy for size, the Tier 1 capital ratio of each bank and a quarter-time fixed effect. Data on market equity are from CRSP. All other data are from the FR Y-9C reports. Return on equity over the next year is defined as book net income over the next four quarters divided by book equity in the current quarter. The two-year ahead loan provision rate is calculated as the ratio of eight quarter ahead quarterly loan provisions divided over total loans. The share of delinquent loans is the ratio of 30 days or more past due loans plus loans in non-accrual over total loans. The net charge-off rate is calculated as the difference between loan charge-offs over the next quarter and loan recoveries over the next quarter, divided by total loans this quarter.

Next, we formally show that variation in the cross-section of Tobin's Q reflects differences in the information content of market and book equity values. If market equity values contain more information about bank profitability and credit losses than book equity values, then we expect for Tobin's Q to predict future bank profits and loan losses even after controlling for book equity. In Figure A.2, we show binned scatter plots of logged outcomes on the log market-to-book equity ratio. The plots control for time fixed effects, the Tier 1 regulatory capital ratio, and log book equity.[56] The top left panel shows the log return on equity over the next year plotted against the log market-to-book ratio. Banks with higher market-to-book ratios earn higher future profits. A bank with a lower Tobin's Q today is also more likely to have higher loan loss previsions even eight quarters ahead (top right panel). Banks with higher market-to-book ratios also have a lower share

and thus taken off the books, which shows up as charge-offs. Occasionally, the bank can recover the asset later.

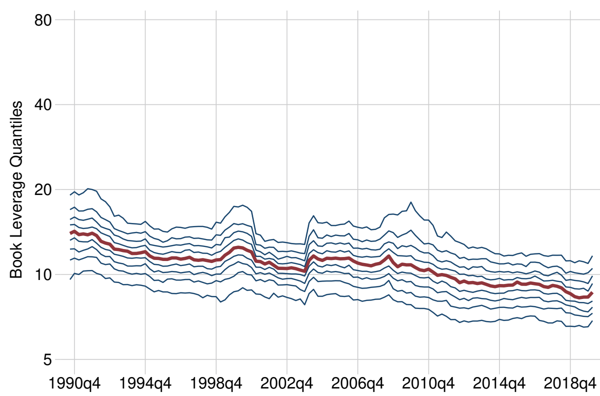
[56] To control for log book equity, the left and right-hand side variables are residualized on log book equity, and then the mean of each variable is added back to maintain the centering. It is important to control for log book equity to prevent spurious results due to ratio bias (see Kronmal, 1993).

of delinquent loans (bottom left panel) and a lower future net charge-off rate on their loans (bottom right). Thus, Tobin's Q predicts future book realized profits and actual loan losses beyond what is reflected in book values, suggesting that book values account for loan losses only very slowly. This is consistent with the fact that book equity did not decline during the crisis, despite widespread issues in credit markets. Note that discount rate variations affect most banks similarly and are therefore unlikely to drive these cross-sectional results. Indeed, our results suggest that banks with lower profitability and more delinquencies have lower Tobin's Q, and that Tobin's Q predicts future loan write-downs and future profitability.

A.2.4 Book Leverage Distribution

Figure A.3 shows that the distribution of book leverage is much less dispersed relative to the market leverage distribution. In fact, it is also relatively stable: the 90th percentile and 10th percentile of the book leverage distribution differed only by a factor of two. This stands in stark contrast to the market leverage distribution in Panel B of Figure 3.

Figure A.3: Quantiles of Book Leverage

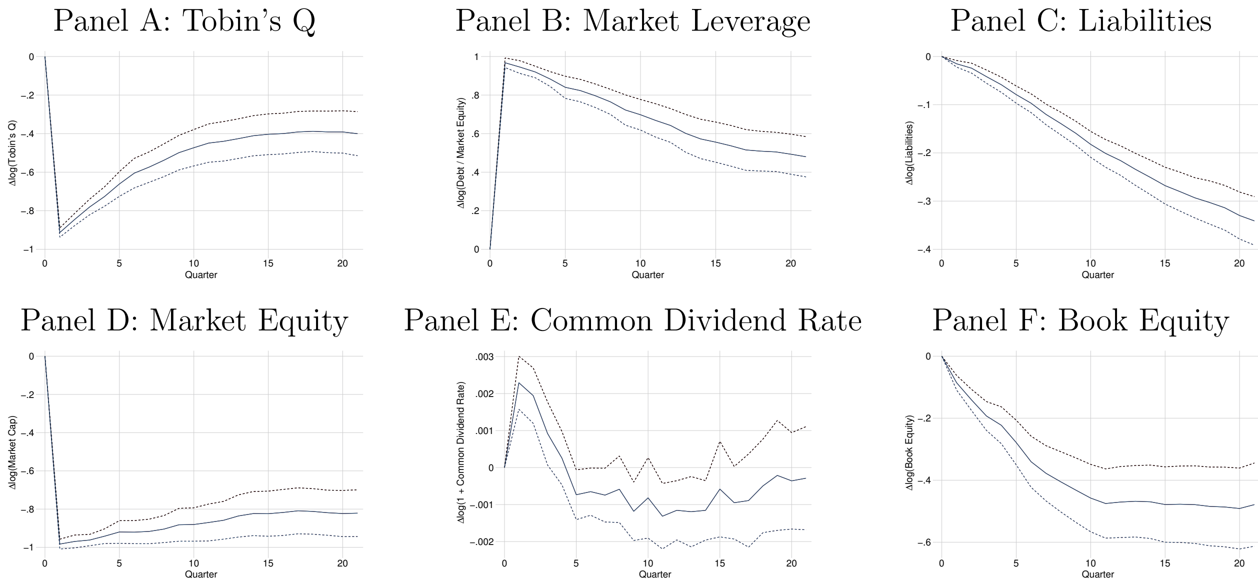


Notes: This figure shows the quantiles of book leverage for BHCs on a log scale. Book data (book equity and liabilities) comes from the FR Y-9C. Book leverage is computed as (liabilities + book equity)/book equity. The median value is plotted in red. Each tenth percentile is plotted in blue.

A.2.5 Impulse Response of Banks

In Figure A.4, we show the impulse response of bank holding companies to a negative net worth shock. This figure is the same as Figure 4 in the main text, but it includes additional variables. In particular, panel F shows the impulse response of book equity, which has a small response on impact on impact and then slowly declines over time, reflecting the gradual recognition of losses on the books. Panel E shows the impulse response of the common dividend rate. We define the log common dividend rate as log(1 + Common Dividends/Market Capitalization). The dividend rate temporarily rises after the shock before returning to a level close to its pre-shock level. This initial rise is driven by the mechanical impact of falling market equity, since market capitalization is in the denominator of the dividend rate. Thus, the response of the dividend rate implies that dividends slowly adjust down after the shock.

Figure A.4: Estimated Impulse Responses



Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated percent impulse response to a 1% negative return shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log Tobin's Q (Panel A), log market leverage (Panel B), log liabilities (Panel C), log market capitalization (Panel D), the logged common dividend rate (Panel E), and log book equity (Panel F). Market leverage is defined as (Liabilities/Market Capitalization). The logged common dividend rate is defined as log(1 + Common Dividends/Market Capitalization).

A.3 Stylized Facts: Non-Financial Firms

To compare banks to non-financial firms, we use merged data from CRSP-Compustat, excluding firms in finance, insurance, and real estate. We recompute our main results for these non-financial firms; the results are in Figure A.5, A.6, A.7, and A.8. In each figure, we follow the same empirical strategy as in the main text of the paper, but apply these methods to the CRSP-Compustat data for non-financials.

Although our paper does not offer a theory of the behavior of non-financial firms, the regulatory enivironment implies differences in the dynamic optimization problem faced by banks vs. non-financials. In particular, banks have strong incentives to be highly levered, which is held in check by regulatory constraints on their book leverage. This creates the dynamic considerations emphasized by our Q-theory.

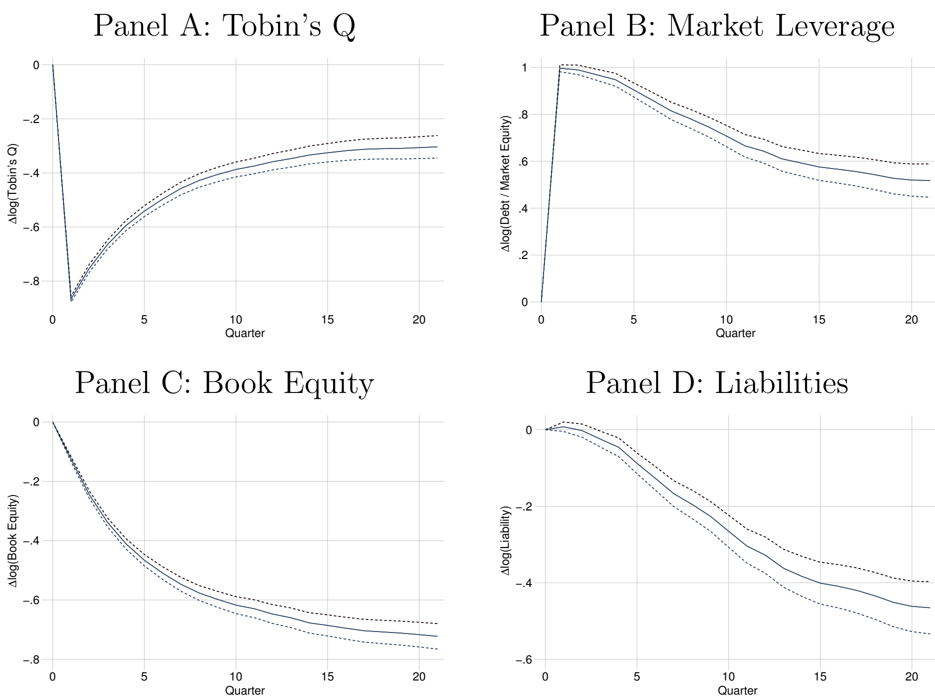
In contrast, these issues are not relevant for most non-financials, who largely hold low leverage and do not face such regulations. Instead, we find that some features of our Q-theory are relevant for non-financial firms, while other features are unique to banks. In particular, the evidence suggests that although historical-cost accounting causes book values at non-financials to lag fundamentals, similarly to banks, this does not interact with those firms' leverage decision, since book leverage regulations are not an important constraint for non-financials. Non-financials do appear to gradually delever in response to net worth shocks, but this is a feature of many dynamic trade-off theory models. Our Q-theory combines a model of accounting with a theory of bank leverage choice; for non-financial firms only the model of accounting is likely to be relevant.

Book values for non-financial firms are often based on historical cost, and thus will lag fundamental values, much like in the banking sector. Thus in Figure A.6, we find that the market-to-book ratio is predictive of future profits, like in the banking sector (Fact 2). Moreover, our estimated impulse responses (Figure A.8; Fact 4) for non-financial firms also find that book equity only slowly reflects net worth shocks, which are (by construction) immediately reflected in market values.

However, non-financial firms face a very different environment when it comes to leverage: they do not have the same incentive to lever up as banks, and are generally not constrained by regulations on leverage. As a result, we do not see the same time series patterns of Tobin's Q and leverage for non-financials as we see for financial firms. Although Tobin's Q fell during the financial crisis for non-financial firms, the drop is smaller than the drop experienced by the banking sector, and is small relative to other fluctuations for non-financials (Figure A.5; Fact 1). Moreover, leverage is low for non-financials, and the distribution of leverage and Tobin's Q is relatively stable throughout the crisis (Figure A.7; Fact 3).

In summary, we find that some of our facts (Facts 2 and 4) are similar for financial and non-financial firms, while others are quite different (Facts 1 and 3).

Figure A.8: Estimated Impulse Responses for Non-Financial Firms



Notes: These figures show the estimated percent impulse response to a 1% negative return shock, for non-financial firms. These figures are the counterpart to Figure 4 in the body of the paper, and are constructed in the same way, but for our sample of non-financial firms in CRSP-Compustat. The y-axis of our plots shows the contemporaneous response ($-\beta_0$) as quarter 1, the cumulative response one quarter later ($-\beta_0 - \beta_1$) as quarter 2, and so on. Dashed lines denote the 95% confidence interval. Standard errors are clustered by firm. The panels display the impulse responses of log Tobin's Q (Panel A), log market leverage (Panel B), log book equity (Panel C), and log liabilities (Panel D). Market leverage is defined as (Liabilities/Market Capitalization).

B Details on Impulse Response Function Estimation

B.1 Risk Adjustment

In this subsection, we describe our impulse response estimation procedure in more detail, and establish that it consistently estimates the impulse response to an idiosyncratic net worth shock.

For our main impulse response results, we wish to use risk-adjusted returns, rather than raw returns. More formally, we assume that the market returns of bank i at time t are given by

$$\underbrace{r_{it}}_{\text{Raw Return}} - \underbrace{r_t^f}_{\text{Risk-Free Rate}} = \alpha_i + \underbrace{X_t'}_{\text{factors}} \underbrace{\beta_i}_{\text{loadings}} + \underbrace{\varepsilon_{it}}_{\text{Idiosyncratic Component}}$$

All returns are logged, e.g. r_{it} refers to $\log(1 + \text{Raw Bank Return})$. We wish to isolate variation in the idiosyncratic shocks, ε_{it}, and use this variation to estimate the impulse responses.

A natural, but naive, approach would be to estimate the above model for each bank i using OLS, and then use the estimated residuals, $\hat{\varepsilon}_{it}$, as the regressors in the impulse response estimation. The problem here is that it induces bias: $\hat{\varepsilon}_{it}$ is a noisy measure of the true regressor ε_{it}, which leads to bias as long as T is finite (the bias will shrink as T grows large, because $\hat{\varepsilon}_{it}$ will converge to the true ε_{it}).

Fortunately, there is a simple solution: we estimate $\hat{\varepsilon}_{it}$ using OLS, and then we use $\hat{\varepsilon}_{it}$ as an *instrument* for the unadjusted return, r_{it}. Since our main regressions use contemporaneous returns and twenty lags, this means we use contemporaneous $\hat{\varepsilon}_{it}$ and twenty lags of $\hat{\varepsilon}_{it}$ as instruments. Instrumental variables does not suffer from the same problem of bias under classical measurement error. Instead, to get identification under the assumed model for returns, we need our instrument to be correlated with the "good variation", ε_{it}, and uncorrelated with the "bad variation," $\alpha_i + X_t'\beta_i$. This is exactly what we get when we estimate a bank-level OLS regression of returns on factors, in order to get $\hat{\varepsilon}_{it}$. Although our application of the risk-adjustment to this setting is novel, this procedure (residualizing a potential shock on controls, and using the residual as an instrument) is similar to that of Kanzig (2021), who performs a related procedure to identify oil supply shocks.

Our instrumental variables strategy will give us a consistent estimator of the true impulse response, under the assumption that we have the correct model of returns. Since the OLS regression estimating $\hat{\varepsilon}_{it}$ is conducted at the bank level, which mechanically creates correlation in the instrument at the bank level, we cluster our standard errors at the bank level.

To summarize, our procedure consists of two steps:

1. Estimate a factor model, $r_{it} - r_t^f = \alpha_i + X_t'\beta_i + \varepsilon_{it}$, for each bank i, and generate estimated idiosyncratic shocks, $\hat{\varepsilon}_{it}$.

2. Estimate the IV regression $\Delta \log(y_{i,t}) = \alpha_t + \sum_{h=0}^{k} \beta_h \cdot r_{i,t-h} + \psi_{i,t}$, using the estimated vector of idiosyncratic shocks, $(\hat{\varepsilon}_{i,t-h})_{h=0}^{k}$ as instruments for returns, $(r_{i,t-h})_{h=0}^{k}$.

Proof of Consistency for Risk Adjustment Procedure Below, we provide a formal justification of this procedure, and provide the conditions under which it will provide us consistent estimates as $N \to \infty$. Note the importance of focusing on $N \to \infty$ asymptotics, rather than also assuming that $T \to \infty$: if we had a large number of time periods, then the measurement error in $\hat{\varepsilon}_{it}$ would be small, and we would not need to use the IV procedure (we could just use $\hat{\varepsilon}_{it}$ directly). However, there is meaningful estimation error in $\hat{\varepsilon}_{it}$, and so the IV correction is necessary to ensure that $\hat{\beta} \xrightarrow{p} \beta$.

For expositional simplicity, we will focus on the univariate case ($k = 0$). The extension to include lags of ε_{it} is analagous, and is omitted for brevity. Assume the following:[57]

$$y_{it} = \alpha_t + \beta\varepsilon_{it} + u_{it}$$
$$r_{it} = X_{it}'\gamma_i + \varepsilon_{it}$$

[57] Note that here, for simplicity, we write r_{it}, but in our implementation we use $r_{it} - r_t^f$ to estimate $\hat{\varepsilon}_{it}$. The results here will go through for $r_{it} - r_t^f$, because the time fixed effect in the main regression will absorb r_t^f.

We will make two key assumptions. We will assume that $\mathbb{E}[\varepsilon_{it} \mid X] = 0$: this means that we have correctly specified the factor model of bank returns, r_{it}. Moreover, we will assume $\mathbb{E}[\varepsilon_{is} u_{it} \mid X] = 0 \, \forall s, t$. This assumption means that, regardless of the factor draws X, we will still have that ε and u are orthogonal. This latter assumption is thus a slight strengthening of the typical IV orthogonality condition.

We cannot just regress y_{it} on r_{it}, because X_{it} may be correlated with the error term, u_{it}. We want to isolate the effect of ε_{it}. To get an estimate of ε_{it}, we will first estimate $\hat{\varepsilon}_{it}$ using an OLS regression of r_{it} on X_{it}. This yields:

$$\begin{aligned}
\hat{\varepsilon}_{it} &= r_{it} - X'_{it}\hat{\gamma}_i \\
&= r_{it} - X'_{it}\left(\frac{1}{T}\sum_s X_{is}X'_{is}\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}r_{is}\right) \\
&= r_{it} - X'_{it}\left(\frac{1}{T}\sum_s X_{is}X'_{is}\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\left(X'_{is}\gamma_i + \varepsilon_{is}\right)\right) \\
&= \varepsilon_{it} - X'_{it}\left(\frac{1}{T}\sum_s X_{is}X'_{is}\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\varepsilon_{is}\right)
\end{aligned}$$

A naive approach would be to use $\hat{\varepsilon}_{it}$ as our regressor in an OLS regression of y_{it} on $\hat{\varepsilon}_{it}$. However, this will in general not yield consistent estimates of β either, because $\hat{\varepsilon}_{it}$ is a noisy measure of the true ε_{it}. Instead, we will use $\hat{\varepsilon}_{it}$ as an instrument for r_{it}. That is, we will run the regression:

$$y_{it} = \alpha_t + \beta r_{it} + \psi_{it}$$

and we will use $\hat{\varepsilon}_{it}$ as an instrument for the endogenous regressor r_{it}. Of course, our structural equation has β as the effect of ε_{it}, not r_{it}. We can rewrite the original structural equation:

$$\begin{aligned}
y_{it} &= \alpha_t + \beta\left(r_{it} - X'_{it}\gamma_i\right) + u_{it} \\
&= \alpha_t + \beta r_{it} + \underbrace{u_{it} - \beta X'_{it}\gamma_i}_{\psi_{it}}
\end{aligned}$$

Thus, our IV regression will identify the correct β as long as our instrument, $\hat{\varepsilon}_{it}$, is uncorrelated with the residual, $\psi_{it} = u_{it} - \beta X'_{it}\gamma_i$.

We will now show that $\mathbb{E}[\hat{\varepsilon}_{it}(u_{it} - \beta X'_{it}\gamma_i)] = 0$. We have $\mathbb{E}[\hat{\varepsilon}_{it}(u_{it} - \beta X'_{it}\gamma_i)] = \mathbb{E}[\hat{\varepsilon}_{it}u_{it} - \hat{\varepsilon}_{it}\beta X'_{it}\gamma_i]$.

We will address each component in turn:

$$
\begin{aligned}
\mathbb{E}\left[\hat{\varepsilon}_{it}u_{it}\right] &= \mathbb{E}\left[\varepsilon_{it}u_{it} - X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\varepsilon_{is}\right)u_{it}\right] \\
&= \mathbb{E}\left[-X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\varepsilon_{is}\right)u_{it}\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[-X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\varepsilon_{is}\right)u_{it} \mid X\right]\right] \\
&= \mathbb{E}\left[-X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\frac{1}{T}\sum_s X_{is}\mathbb{E}\left[(\varepsilon_{is}u_{it}) \mid X\right]\right] \\
&= \mathbb{E}\left[-X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\frac{1}{T}\sum_s X_{is}\cdot 0\right] \\
&= 0
\end{aligned}
$$

The other component is:

$$
\begin{aligned}
\mathbb{E}\left[-\hat{\varepsilon}_{it}\beta X_{it}'\gamma_i\right] &= \mathbb{E}\left[X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\varepsilon_{is}\right)\beta X_{it}'\gamma_i\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\varepsilon_{is}\right)\beta X_{it}'\gamma_i \mid X\right]\right] \\
&= \mathbb{E}\left[X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\mathbb{E}\left[(\varepsilon_{is}) \mid X\right]\right)\beta X_{it}'\gamma_i\right] \\
&= \mathbb{E}\left[X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\cdot 0\right)\beta X_{it}'\gamma_i\right] \\
&= 0
\end{aligned}
$$

The sum of these components is zero, which proves that our instrument is orthogonal to the residual. Thus, as long as the instrument is relevant (the instrument is correlated with returns), we will consistently estimate β as $N \to \infty$.

B.2 Inferring Impulse Responses from Coefficients

We can infer the impulse response from the coefficients of our model. In particular, the impulse response over k quarters will be equal to $\sum_{h=0}^{k}\beta_h$. To make this clear, we provide a short inductive proof.

We define the impulse response as $\mathbb{E}\left[\log y_{i,t+k} \mid \varepsilon_{i,t} = 1\right] - \mathbb{E}\left[\log y_{i,t+k}\right]$. For $t < 0$, the impulse response is zero, since the bank does not respond to shocks that have not happened yet (shocks are unanticipated). For $t \geq 0$, the impulse response can be backed out by induction.

$$\underbrace{\mathbb{E}\left[\log y_{i,t+k} \mid \varepsilon_{i,t}=1\right]-\mathbb{E}\left[\log y_{i,t+k}\right]}_{\text{Impulse Response for horizon } k}=\underbrace{\mathbb{E}\left[\log y_{i,t+k-1} \mid \varepsilon_{i,t}=1\right]-\mathbb{E}\left[\log y_{i,t+k-1}\right]}_{\text{Impulse Response for horizon } k-1}$$
$$+\underbrace{\mathbb{E}\left[\log y_{i,t+k}-\log y_{i,t+k-1} \mid \varepsilon_{i,t}=1\right]-\mathbb{E}\left[\log y_{i,t+k}-\log y_{i,t+k-1}\right]}_{\text{Impulse response between horizons } k-1 \text{ and } k}$$

Using stationarity, we have:

$$\begin{aligned}
&\mathbb{E}\left[\log y_{i,t+k}-\log y_{i,t+k-1} \mid \varepsilon_{i,t}=1\right]-\mathbb{E}\left[\log y_{i,t+k}-\log y_{i,t+k-1}\right] \\
&=\mathbb{E}\left[\log y_{i,t}-\log y_{i,t-1} \mid \varepsilon_{i,t-k}=1\right]-\mathbb{E}\left[\log y_{i,t}-\log y_{i,t-1}\right]
\end{aligned}$$

Using our regression equation, we know that this equals

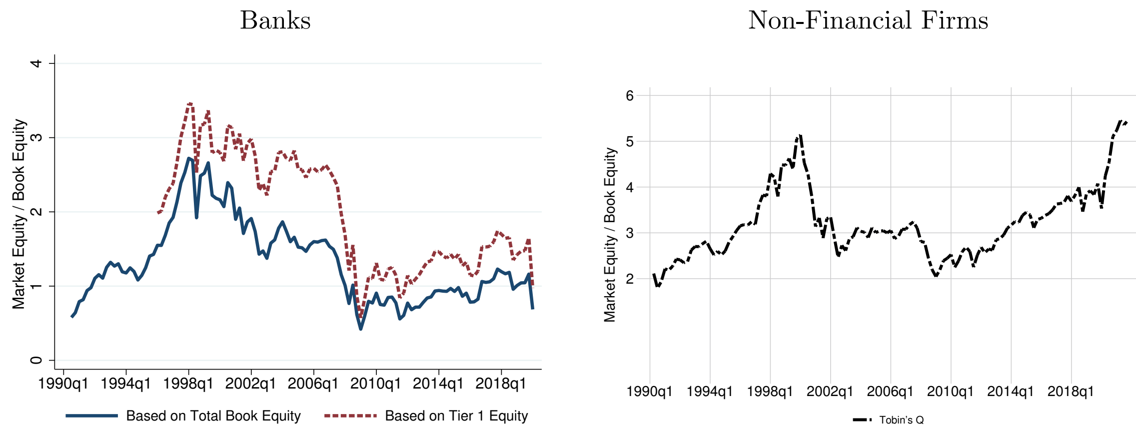
$$\sum_{h=0}^{k} \beta_h \left(\mathbb{E}\left[\varepsilon_{i,t-h} \mid \varepsilon_{i,t-k}=1\right]-\mathbb{E}\left[\varepsilon_{i,t-h}\right]\right)$$

Since the shocks are mean independent, we know that all of these expectation differences are zero, except for the one for $h=k$. Thus, we have:

$$\mathbb{E}\left[\log y_{i,t+k}-\log y_{i,t+k-1} \mid \varepsilon_{i,t}=1\right]-\mathbb{E}\left[\log y_{i,t+k}-\log y_{i,t+k-1}\right]=\beta_k$$

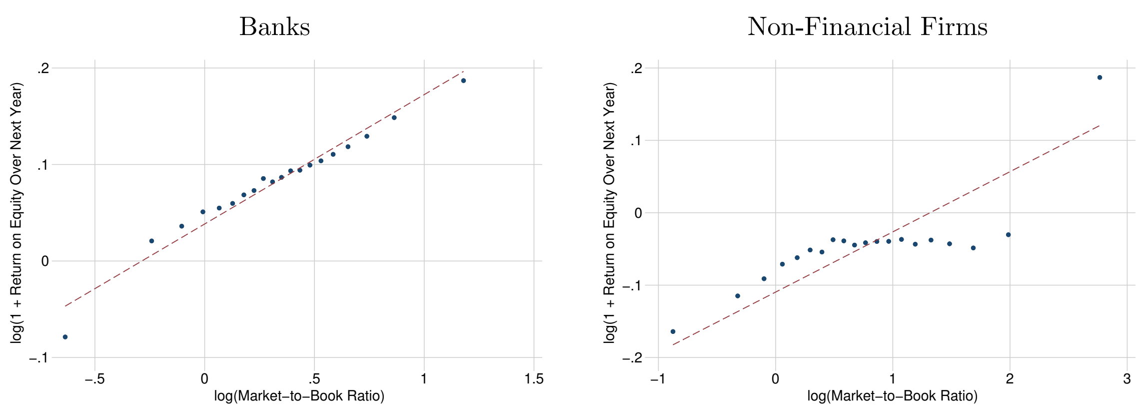
Then, using induction, we find that the impulse response is $\sum_{h=0}^{k} \beta_h$.

Figure A.5: Tobin's Q for Banks and Non-Financial Firms



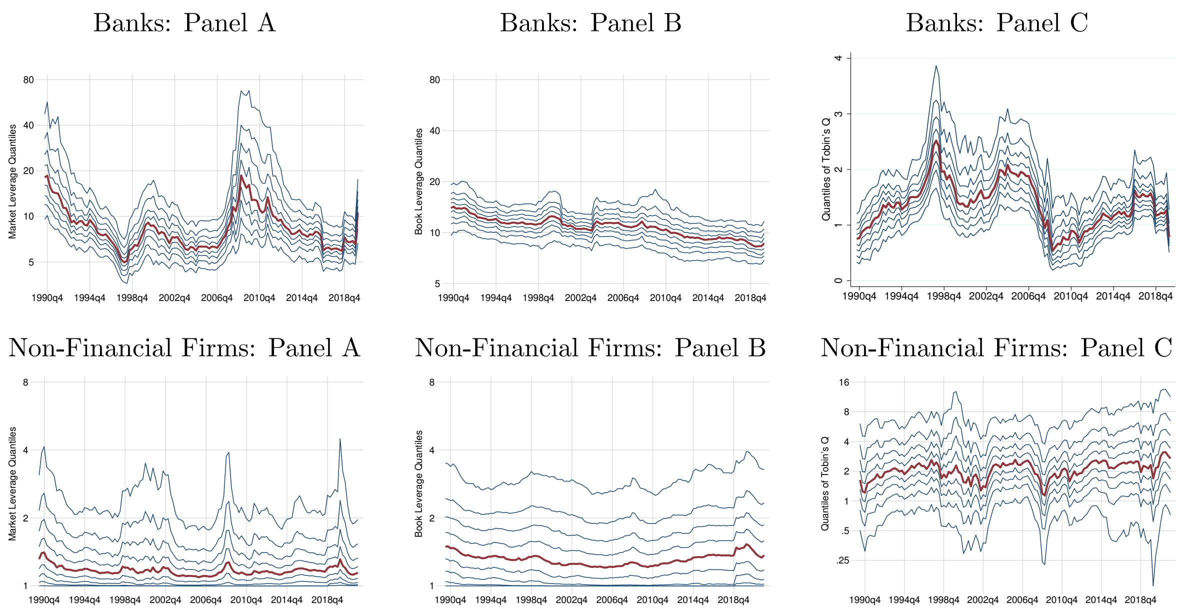
Notes: These figures show data on Tobin's Q panel for an aggregate sample of publicly traded Bank holding companies (left panel) and non-financial firms (right panel). The left panel repeats the figure from the body of the paper. Tobin's Q is the ratio of market equity to book equity and the ratio of market equity to Tier 1 equity capital (only available since 1996). Bank's book equity is from the FR Y-9C, and market equity is from CRSP. The right panel shows results for non-financial firms. Non-financial firm's book equity is from Compustat, and market equity is from CRSP.

Figure A.6: Tobin's Q Predicts ROE for Banks and Non-Financial Firms



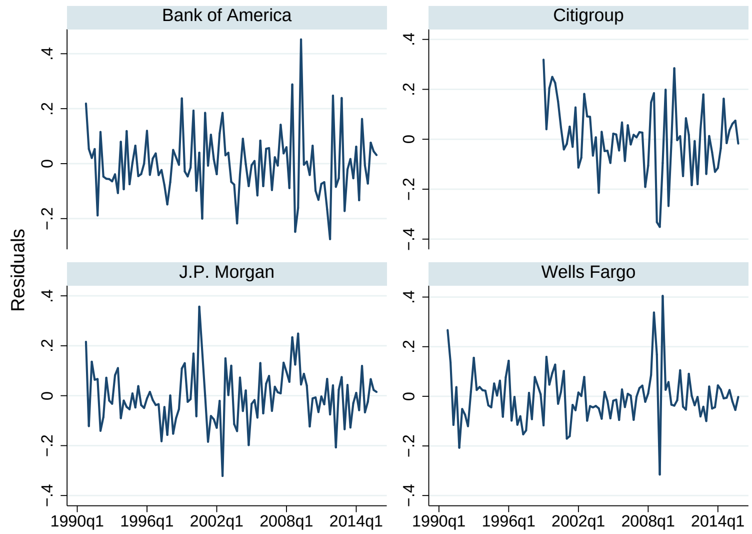
Notes: These figures show cross-sectional binned scatter plots of log outcomes on the log Tobin's Q for BHCs. All plots control for log book equity by residualizing the variables on log book equity, and then adding back the mean of each variable to maintain centering. The left panel shows results for banks, repeating the figure from the body of the paper. Data on market capitalization are from CRSP, and book data are from the FR Y-9C. The right panel shows the same figure but for non-financial firms, with book data from Compustat and market equity from CRSP. ROE over the past year is defined as book net income over the last four quarters divided by book equity four quarters ago; ROE over the next year is defined the one lead of this variable (i.e. profits over the next four quarters divided by current book equity).

Figure A.7: Quantiles of Market Leverage, Book Leverage, and Tobin's Q for Banks and Non-Financial Firms



Notes: This figure shows the quantiles of market leverage (Panel A), book leverage (Panel B), and Tobin's Q (Panel C) on a log scale. The top row shows results for banks, repeating figures from the body of the paper. Book data (liabilities) comes from the FR Y-9C, and market equity data is from CRSP data. The bottom row shows the same figures but for non-financial firms, where book data comes from Compustat and market data comes from CRSP. Market leverage is computed as (liabilities + market equity)/market equity. Book leverage is computed as (liabilities + book equity)/book equity. Tobin's Q is computed as market equity/book equity. The median value is plotted in maroon. Each tenth percentile is plotted in blue.

Figure B.1: Idiosyncratic Shock Series of Big Four Bank Holding Companies



Notes: This figure plots the idiosyncratic shocks (for the Big Four BHCs) used to estimate the impulse response functions. First, we isolate the idiosyncratic component of returns using the factor model, and then we residualize this on time fixed effects.

B.3 Robustness and Validity of Identification Strategy

In this section, we conduct various tests to check the validity of our identification strategy and robustness of our results.

A narrative approach to corroborate the idiosyncratic shocks To provide corroberating evidence of the validity of our identification strategy, we first show that the estimated return shocks do indeed look like idiosyncratic shocks for the four largest banks (Bank of America, J.P. Morgan Chase, Wells Fargo, Citigroup). To construct the idiosyncratic shocks, we regress each bank's market return on the Fama-French three-factor returns and regress the residual further on time fixed effects. The residuals from this regression represent our idiosyncratic shocks.[58] Figure B.1 presents our estimates of the idiosyncratic shocks. They indeed look like white noise and do not seem to be substantially autocorrelated. Note that the time series for Citigroup starts a little later because Citigroup did not exist until 1998 when Traveler's merged with Citicorp.

We also provide narrative support for the idiosyncratic nature of our estimated shocks using an extensive search of newspaper articles for large idiosyncratic shock value estimates.

[58] We are controlling for the time fixed effects, because they are included in the regression we actually run to get the impulse response function.

Table 2: Narrative Support for Idiosyncratic Shocks

Bank Name	Year-Qtr	idiosyncratic shock	Bank specific events
Bank of America	2000q4	-0.200	Sunbeam (which BofA lended to) posted $86M loss. BofA said net charge-offs in Q4 will double. BofA issues warning on $1B uncollectible debt, may miss the December quarter profit forecast by as much as 27%.
	2003q4	-0.218	BofA agrees to pay $47 to buy FIeetBoston Financial "hefty premium" & "could dilute earnings."
	2008q3	0.288	BofA to buy Merrill for $50B (Sept 15)
	2009q2	0.452	Stress test: BofA needs to address $34B capital shortfall, better than expectation.
	2011q4	-0.275	Merrill Lynch has agreed to pay $315 million to end a mortgage-securities lawsuit (Dec 7)
	2012q4	0.248	BofA considered better buy after increase in house prices that (given its portfolio composition) particularly benefited BofA.
Citigroup	1999q1	0.319	Citigroup Profit Fell 53% in 4th period, but still topped analysts' expectations
	1999q3	0.205	Citigroup posts an unexpected increase of 9.3% in net income for second quarter (July 20)
	1999q4	0.250	Citigroup's citibank unit is marketing credit card for the internet to millions
	2000q1	0.226	Citi Intelligent Technology Receives Investment; Dividends increase from $1.05 to $1.20
	2003q4	-0.215	Citi to repay certain funds $16 mln plus interest; Citigroup Asset Management faces federal probe.
	2009q1	-0.351	Citigroup had $2B in direct gross exposure to LyondellBasell Industries, who filed for bankruptcy protection last week. Fitch cuts Citi preferred to junk
	2009q3	0.199	Citi reports profit after gain from Smith Barney. Citigroup's mortgage mitigation rises 29% in second quarter.
	2009q4	-0.267	Citi fined in tax crackdown. Abu Dhabi's sovereign wealth fund is demanding that Citigroup scraps a deal that would see the fund make a heavy loss on a $7.5 billion investment in the bank.
	2010q2	0.285	Citi reported quarterly earnings of $4.4B exceeding expectations
J.P. Morgan Chase	1997q2	-0.182	J.P. Morgan particularly large exposure to 1997 Asian Financial Crisis. https://www.imf.org/external/pubs/ft/wp/1999/wp99138.pdf
	2000q1	0.169	J.P. Morgan told investors on Monday that January and February had topped performance levels seen in the fourth quarter. Dividends increase from $0.2733 to $0.3200 on March 21.
	2000q3	0.357	Chase buying J.P. Morgan.
	2001q2	-0.185	J.P. Morgan Chase disclosed this week that their venture capital portfolios had incurred significant losses.
	2002q3	-0.322	JPMorgan Partners Reports $165M Operating Loss for Q2. J.P. Morgan sees third-quarter shortfall.
	2004q4	-0.198	JPMorgan Chase profit falls 13%.
	2008q3	0.234	J.P. Morgan profit falls 53%, but tops Wall Street target.
	2009q1	0.249	J.P. Morgan net falls sharply, but tops Wall Street view. J.P. Morgan to sell Bear Wagner to Barclays Capital: WSJ
	2012q2	-0.207	J.P. Morgan: London Whales $2 Billion Losses. Two Shareholder Suits Filed Against J.P. Morgan
Wells Fargo	2001q2	-0.161	Wells Fargo disclosed that their venture capital portfolios had incurred significant losses. Wells Fargo to take $1.1 billion charge
	2008q3	0.338	Wells Fargo's net dropped 21% as it set aside $3 billion for loan losses, better than expected. Earnings declined but beat estimates.
	2009q1	-0.315	Wells Fargo posted a surprise $2.55B Q1 loss, later revised to $2.77B. Wells Fargo added a pretax $328.4M impairment of perpetual preferred securities to its fourth-quarter loss.
	2009q2	0.405	Wells Fargo sees record Q1 profit, projections easily exceed expectations (expects earnings of $3 billion).

Table 2 shows that large absolute idiosyncratic shock values are consistent with good or bad bank specific events, such as "Wells Fargo sees record Q1 profit, projections easily exceed expectations," or "Citi fined in tax crackdown." The table shows that large positive or negative idiosyncratic shocks can be corroborated with specific events that appear bank specific, which supports the validity of our identification strategy.

Placebo Tests To test the validity of our identification strategy, we conduct placebo tests where we include ten leads of returns (in addition to the contemporary value and twenty lags as before). If the returns really are unanticipated shocks, then the leading values should not affect current behavior. This is similar to testing for pre-trends. We are testing whether the banks that will experience higher returns in the future are already acting differently today. Overall, the placebo test are encouraging, and suggest that our results are not driven by prior differences in the behavior of banks which experience return shocks.

Identification robustness We provide a few additional pieces of evidence that corroborate the validity and robustness of our identification strategy.

First, we verify that our results are robust to excluding the crisis years 2008 and 2009 from our sample. This rules out the notion that our results are related to specific events during the crisis (e.g. the realization that the government might not guarantee that a bank wouldn't fail, or that this was somehow about exposure to Lehman). Doing this made no noticeable difference to the results, since these years are only a small part of the sample.

We also check whether bank mergers drive the results. To this end, we drop the quarter of the merger as well as the quarter before and after the merger. Again, this made no noticeable difference to the results, since only a small number of observations were dropped.

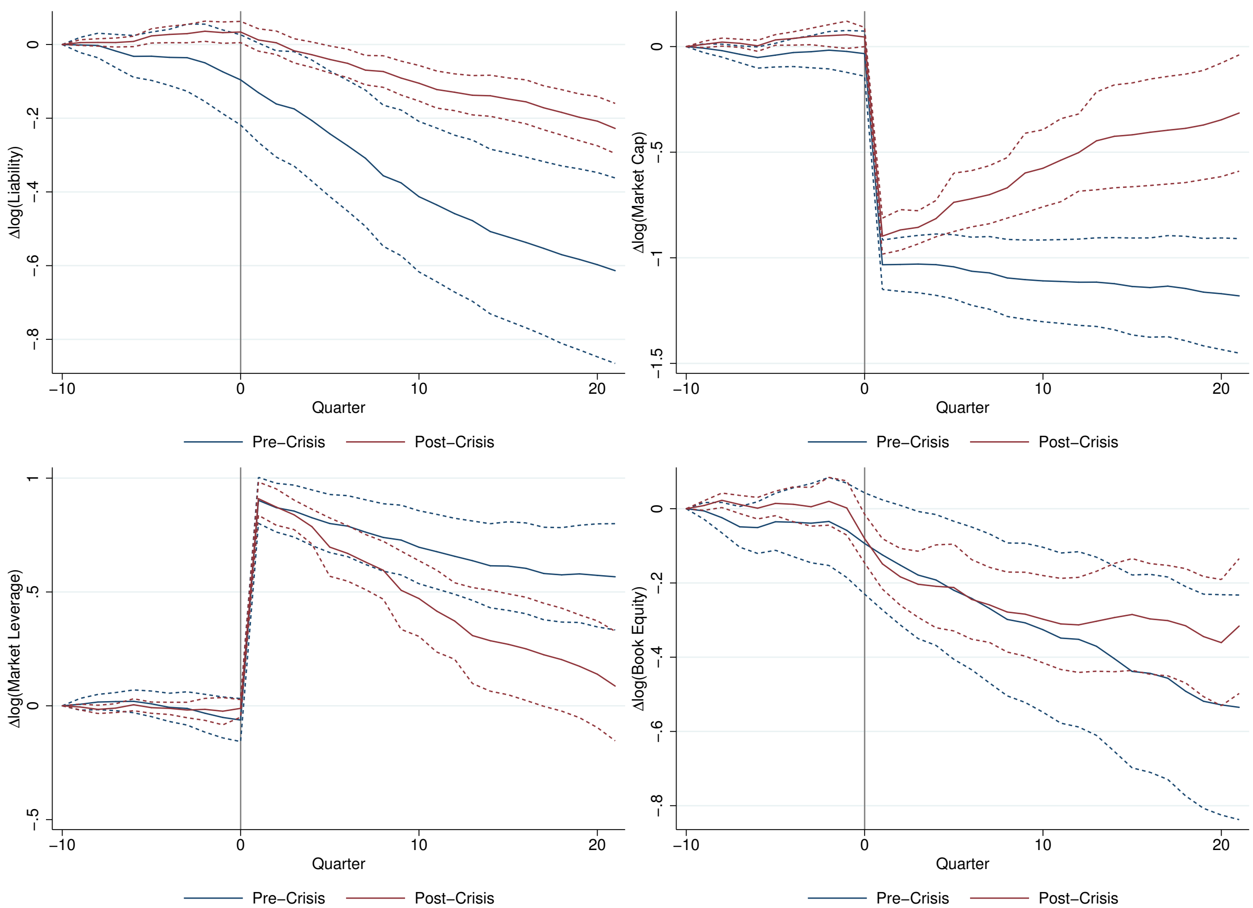
Similarly, we check whether the results are driven by the stress tests performed by banks: these stress tests were implemented after the onset of the crisis, and encouraged or mandated that banks raise additional capital. To show that the stress tests do not drive the results, we drop all banks that ever participated in a stress test (e.g. Bank of America participated in the stress tests, and so we drop Bank of America from our sample in all periods). Again, this makes no noticeable difference to the results, because only a handful of banks in the sample were ever subject to stress tests.

Another potential concern is that the return shocks could be picking up shocks to future investment opportunities, rather than default shocks. To test this concern, we check the response of the liquid assets ratio: if negative return shocks indeed predict lower future investment opportunities rather than current cash flows, we would expect banks to respond to these shocks by moving their portfolio into liquid assets. The results are in Figure B.3. Panel A shows the response of our main measure of the liquidity ratio, which we define as (Cash + Treasury Bills) /Total Assets); within the regression sample, the average liquidity ratio is 5.7%. We find a small, temporary response that begins to reverse after two years. Panel B shows an alternative measure of the liquid assets ratio, defined as (Cash + Federal Funds Sold + Securities Purchased Under Agreement to Resell + Securities)/Total Assets; the average of this liquidity ratio is 28%. The impulse response function has a significant but quantitatively small response: the impulse response implies that a 10% negative shock to market returns would cause the liquidity ratio to rise by just 0.2 percentage points over the course of two years. We take this as evidence against the hypothesis that return shocks reflect shocks to investment opportunities.

B.4 Heterogeneity

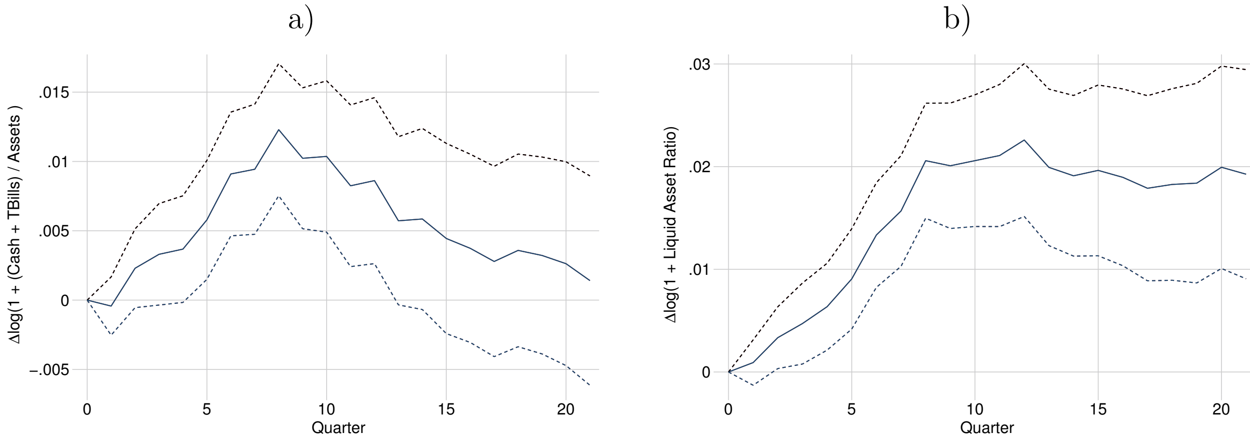
We explore heterogeneity in impulse response functions by dividing banks into two groups based on a variable, and estimating impulse responses separately for each group. We divide banks by size

Figure B.2: Estimated Impulse Responses for Stock Variables (Risk-Adjusted, with Placebo)



Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities + Market Capitalization)/Market Capitalization) are extremely similar).

Figure B.3: Estimated Impulse Responses of the Liquidity Ratio



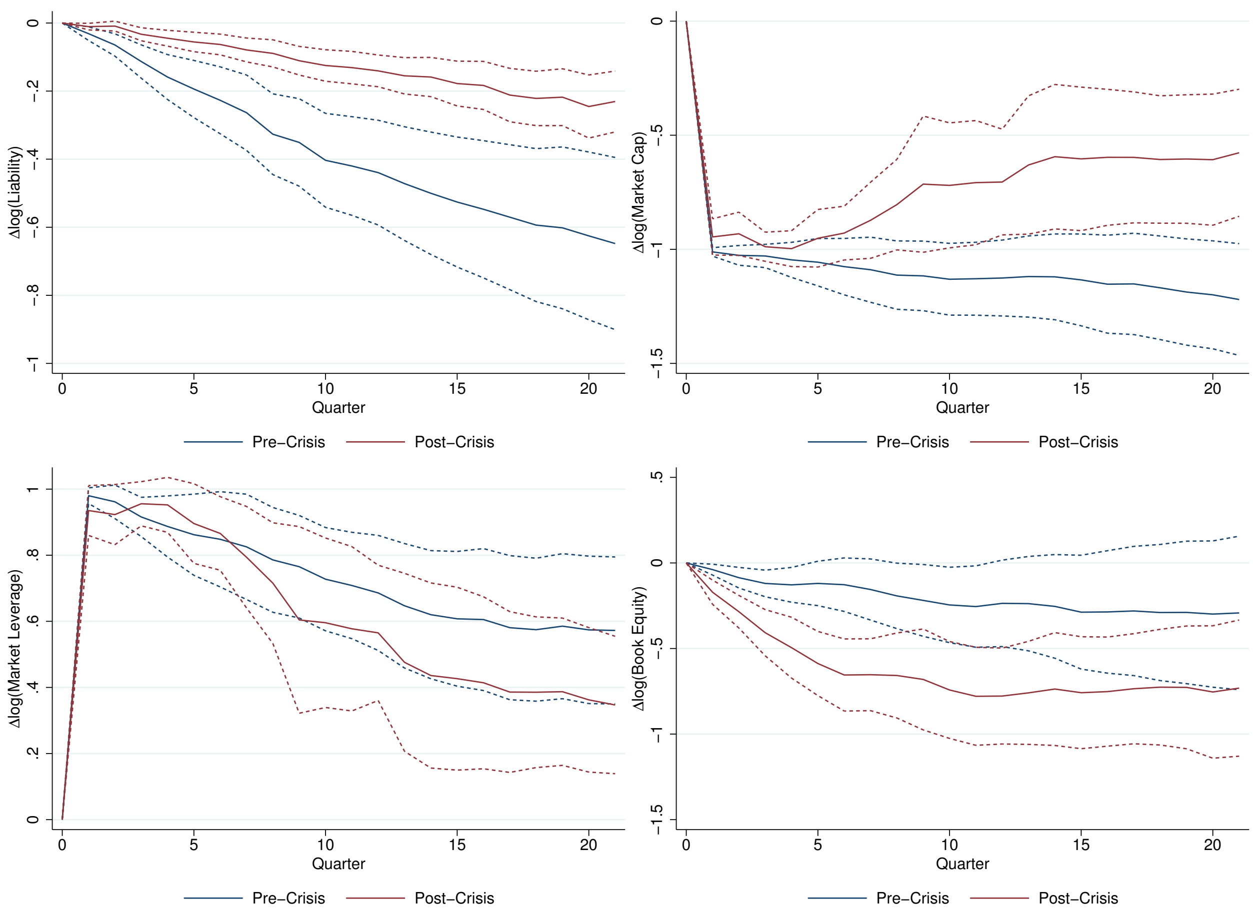
Notes: This figure shows the estimated impulse response function for BHCs to a 1% negative return shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. Panel A shows our main measure of the liquid assets ratio, defined as $\log((\text{Cash} + \text{Treasury Bills})/\text{Total Assets})$. Panel B shows results for our alternative, broader measure of the liquid assets ratio, defined as $\log((\text{Cash} + \text{Fed Funds Sold} + \text{Securities Purchased Under Agreement to Resell} + \text{Securities})/\text{Total Assets})$.

(total assets), by trading assets ratio (trading assets as a share of total assets), by the risk-weighted asset ratio (risk-weighted assets as a share of total assets), and by the mortgage ratio (real estate loans as a share of total assets). We use the value of the variable in 2000 Q1 to sort banks into two groups: above-median and below-median. We report the results in this section. Broadly, we do not find strong evidence of differential responses, but we lack statistical power to rule out some meaningful differences.

Since bank size is among the most important differences across different banks, we begin by discussing the results for heterogeneity by size. The results are shown in Figures B.4 and B.5. Visually, these impulse responses look remarkably similar to each other. However, the standard errors are sufficiently large that we cannot rule out meaningful differences in the impulse responses.

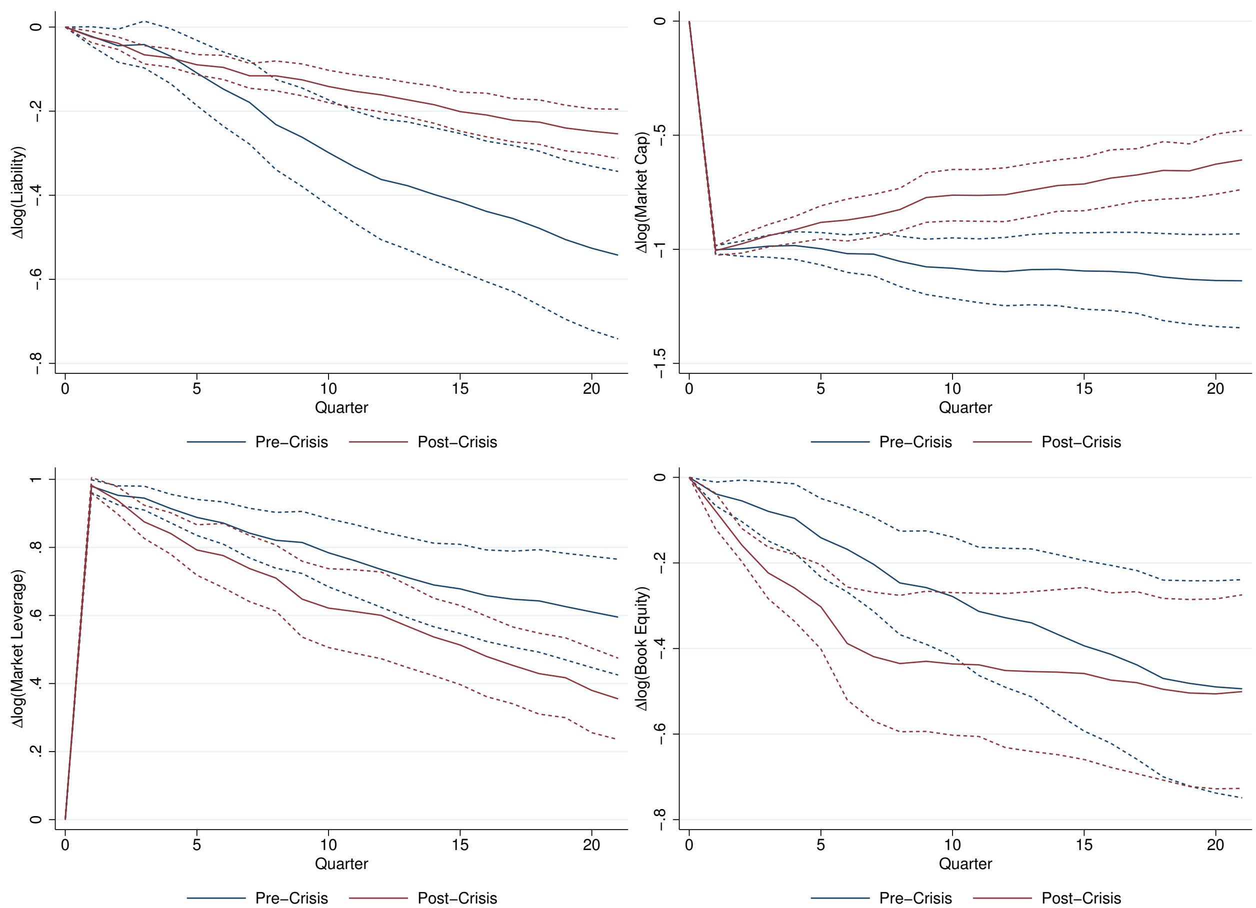
We summarize the results of these impulse responses, as well as of the other potential groupings (by trading assets ratio, risk-weighted assets ratio, and mortgage ratio) in Tables 3, 4, 5, and 6 below. For each grouping, we report the cumulative impulse response for the high and low groups after 10 quarters and after 20 quarters, and we also report the p-value of a test of equality between the impulse responses of the two groups. In a table of 64 tests, only one of the tests rejects the null at the 5% level. As before, we take this to suggest that there is not strong evidence in favor of sizable heterogeneity, but we caution that the standard errors are too large to rule out meaningful heterogeneity.

Figure B.4: Impulse Responses for Small Banks



Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities + Market Capitalization)/Market Capitalization) are extremely similar).

Figure B.5: Impulse Responses for Large Banks



Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities + Market Capitalization)/Market Capitalization) are extremely similar).

Table 3: Heterogeneity in Impulse Responses: Small vs. Large Banks

		Response After 10 Quarters			Response After 20 Quarters		
		Small	Large	p-value on Equality	Small	Large	p-value on Equality
Market Equity	Pre-Crisis	-1.13	-1.09	0.75	-1.22	-1.14	0.61
		(0.08)	(0.07)		(0.12)	(0.11)	
	Post-Crisis	-0.71	-0.76	0.71	-0.58	-0.61	0.84
		(0.14)	(0.06)		(0.14)	(0.07)	
Liabilities	Pre-Crisis	-0.42	-0.33	0.39	-0.65	-0.54	0.52
		(0.07)	(0.07)		(0.13)	(0.10)	
	Post-Crisis	-0.13	-0.15	0.49	-0.23	-0.25	0.67
		(0.02)	(0.02)		(0.05)	(0.03)	
Market Leverage	Pre-Crisis	0.71	0.76	0.59	0.57	0.60	0.87
		(0.08)	(0.05)		(0.11)	(0.09)	
	Post-Crisis	0.58	0.61	0.81	0.35	0.35	0.95
		(0.13)	(0.06)		(0.11)	(0.06)	
Book Equity	Pre-Crisis	-0.25	-0.31	0.68	-0.29	-0.49	0.44
		(0.12)	(0.08)		(0.23)	(0.13)	
	Post-Crisis	-0.78	-0.44	0.04	-0.73	-0.50	0.32
		(0.15)	(0.09)		(0.20)	(0.12)	

Notes: The table compares impulse responses of small vs. large BHCs. BHCs are categorized into the small vs. large group based on their total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre-crisis and post-crisis, to a one unit negative return shock, for small banks. The second column shows the same results, but for large banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-value of a test of equality between the impulse response for small banks vs. large banks. The fourth through sixth columns mirror the first three columns, but examining the cumulative impulse response after 20 quarters.

Table 4: Heterogeneity in Impulse Responses: Low vs. High Trading Asset Ratio

		Response After 10 Quarters			Response After 20 Quarters		
		Low	High	p-value on Equality	Low	High	p-value on Equality
Market Equity	Pre-Crisis	-1.10	-1.20	0.60	-1.15	-1.32	0.54
		(0.05)	(0.19)		(0.07)	(0.28)	
	Post-Crisis	-0.76	-0.57	0.11	-0.60	-0.47	0.33
		(0.09)	(0.08)		(0.10)	(0.09)	
Liabilities	Pre-Crisis	-0.36	-0.36	0.99	-0.56	-0.63	0.78
		(0.05)	(0.14)		(0.08)	(0.22)	
	Post-Crisis	-0.14	-0.15	0.73	-0.24	-0.25	0.91
		(0.02)	(0.04)		(0.03)	(0.05)	
Market Leverage	Pre-Crisis	0.74	0.84	0.35	0.59	0.70	0.50
		(0.05)	(0.10)		(0.07)	(0.14)	
	Post-Crisis	0.63	0.42	0.08	0.37	0.22	0.25
		(0.08)	(0.09)		(0.08)	(0.09)	
Book Equity	Pre-Crisis	-0.25	-0.42	0.35	-0.28	-0.75	0.17
		(0.07)	(0.17)		(0.13)	(0.32)	
	Post-Crisis	-0.62	-0.45	0.39	-0.66	-0.54	0.64
		(0.10)	(0.16)		(0.13)	(0.20)	

Notes: The table compares impulse responses of low vs. high trading asset ratio BHCs. BHCs are categorized into the low vs. high group based on their trading assets as a share of total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre-crisis and post-crisis, to a one unit negative return shock, for low banks. The second column shows the same results, but for high banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-value of a test of equality between the impulse response for low vs. high banks. The fourth through sixth columns mirror the first three columns, but examining the cumulative impulse response after 20 quarters.

Table 5: Heterogeneity in Impulse Responses: Low vs. High Risk-Weighted Asset Ratio

		Response After 10 Quarters			Response After 20 Quarters		
		Low	High	p-value on Equality	Low	High	p-value on Equality
Market Equity	Pre-Crisis	-1.09	-1.15	0.59	-1.10	-1.22	0.43
		(0.07)	(0.08)		(0.11)	(0.12)	
	Post-Crisis	-0.72	-0.79	0.64	-0.52	-0.66	0.44
		(0.12)	(0.09)		(0.14)	(0.11)	
Liabilities	Pre-Crisis	-0.29	-0.41	0.21	-0.47	-0.66	0.20
		(0.06)	(0.07)		(0.10)	(0.11)	
	Post-Crisis	-0.13	-0.17	0.17	-0.25	-0.25	0.97
		(0.03)	(0.02)		(0.05)	(0.03)	
Market Leverage	Pre-Crisis	0.80	0.73	0.47	0.63	0.56	0.59
		(0.07)	(0.06)		(0.09)	(0.10)	
	Post-Crisis	0.59	0.62	0.82	0.27	0.41	0.31
		(0.11)	(0.09)		(0.11)	(0.09)	
Book Equity	Pre-Crisis	-0.19	-0.35	0.23	-0.24	-0.45	0.40
		(0.10)	(0.09)		(0.16)	(0.18)	
	Post-Crisis	-0.49	-0.74	0.17	-0.51	-0.81	0.23
		(0.09)	(0.16)		(0.11)	(0.23)	

Notes: The table compares impulse responses of low vs. high risk-weighted asset ratio BHCs. BHCs are categorized into the low vs. high group based on their risk-weighted assets as a share of total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre-crisis and post-crisis, to a one unit negative return shock, for low banks. The second column shows the same results, but for high banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-value of a test of equality between the impulse response for low vs. high banks. The fourth through sixth columns mirror the first three columns, but examining the cumulative impulse response after 20 quarters.

Table 6: Heterogeneity in Impulse Responses: Low vs. High Mortgage Ratio

		Response After 10 Quarters			Response After 20 Quarters		
		Low	High	p-value on Equality	Low	High	p-value on Equality
Market Equity	Pre-Crisis	-1.04	-1.21	0.17	-1.07	-1.27	0.26
		(0.05)	(0.11)		(0.07)	(0.17)	
	Post-Crisis	-0.75	-0.75	0.98	-0.61	-0.56	0.79
		(0.13)	(0.08)		(0.17)	(0.09)	
Liabilities	Pre-Crisis	-0.28	-0.46	0.11	-0.45	-0.73	0.09
		(0.05)	(0.10)		(0.08)	(0.15)	
	Post-Crisis	-0.17	-0.11	0.09	-0.28	-0.19	0.13
		(0.02)	(0.02)		(0.05)	(0.03)	
Market Leverage	Pre-Crisis	0.76	0.75	0.92	0.62	0.54	0.55
		(0.06)	(0.07)		(0.08)	(0.11)	
	Post-Crisis	0.59	0.64	0.72	0.34	0.37	0.81
		(0.11)	(0.09)		(0.13)	(0.07)	
Book Equity	Pre-Crisis	-0.20	-0.36	0.32	-0.27	-0.42	0.63
		(0.07)	(0.15)		(0.09)	(0.30)	
	Post-Crisis	-0.66	-0.56	0.57	-0.70	-0.59	0.65
		(0.10)	(0.14)		(0.13)	(0.19)	

Notes: The table compares impulse responses of low vs. high mortgage ratio BHCs. BHCs are categorized into the low vs. high group based on their real estate loans as a share of total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre-crisis and post-crisis, to a one unit negative return shock, for low banks. The second column shows the same results, but for high banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-value of a test of equality between the impulse response for low vs. high banks. The fourth through sixth columns mirror the first three columns, but examining the cumulative impulse response after 20 quarters.

C General Equilibrium and Derivation of Social Welfare Function

In this section, we embed our Q theory of banks into a general equilibrium setting. The purpose is to derive a welfare function in order to produce normative implications. The non-financial sector is akin to the two-sector models of He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014). The key distinction is that we introduce Poisson capital destruction events and bank liquidations. We furthermore consider a single household that owns all firms in the economy.

C.1 Environment

A representative risk-neutral household holds wealth in bank equity and firms. There is a homogeneous good produced with capital in two productive sectors. Capital is freely mobile across sectors. One sector (banked firms) produces goods borrowing loans from banks to buy capital. These banked firms operate part of the capital stock and are financed entirely through banks. Capital operated by banked firms yield an immediate return A^L. The rest of the capital stock is operated by non-banked firms that are directly owned by households. The output of non-banked firms is A^D per unit of capital, and they can also supply deposits to banks. Banks are the entities encountered in the text.

Each capital unit depreciates at rate δ. In addition, the capital operated by banked firms can be destroyed at rate $\sigma\varepsilon$. From an aggregate perspective $\sigma\varepsilon$ is like additional depreciation in the banked sector. We assume that: $A^L - \sigma\varepsilon > A^D$ so that, considering depreciation, it is economically efficient to allocate capital to the banked sector. Capital is fully reversible and investment transforms one unit of goods into capital. The price of capital is, therefore, equal to one unit of goods. Loans are fully collateralized by the capital bought by banked firms. The capital destruction shocks lead to loan defaults as in the model.

Notation. We use script variables to denote variables aggregated across banks. For example, W represents the equity in a given bank whereas $\mathcal{W}$ is the aggregate equity of all banks.

Social Costs of Bank Liquidations. To relate capital destruction to loan default events, we assume banks do not diversify loans across banked firms. For simplicity, we assume each bank lends to a single firm—we can generalized this to multiple firms with correlated shocks. Thus, σ can be viewed as the arrival rate of a capital destruction event that affects the customer of a bank. Under this assumption, as in the body of the text, σ captures the intensity of a default event at a given bank (and its firm) and ε the share of the capital that gets destroyed. When there is no liquidation of the bank, the fraction $(1-\varepsilon)$ is the recovery of the collateral that backs the bank loans, which is entirely seized by the bank. Thus, $\varepsilon = 1 - (1-\varepsilon)$ is the loan recovery rate.

When banks are liquidated, the bank only recovers a fraction $\psi < 1$ of the capital that backs the loan. Thus, total losses are:

$$(1 - \psi (1-\varepsilon)) = \varepsilon + (1-\psi)(1-\varepsilon).$$

We assume bank liquidations are socially inefficient:

$$A^L - \sigma (\varepsilon + (1-\psi)(1-\varepsilon)) < A^D.$$

That is, whereas economically there should only be $\lambda W \varepsilon$ in bank wealth losses, when banks are liquidated, the restructuring costs amount to $(1-\psi)(1-\varepsilon)\lambda W$ in additional losses. It is convenient to account for the wealth remaining at the bank after the default event $W+\bar{J}^W W$:

$$W + \bar{J}^W(\lambda) W = \underbrace{\psi(1-\varepsilon)\lambda W}_{\text{recovered loans}} - \underbrace{(\lambda-1)W}_{\text{deposits}} = (1-(1-\psi(1-\varepsilon))\lambda) W.$$

This wealth is used to form new banks. Per unit of wealth, the remaining bank wealth is, $1 - (\varepsilon + (1-\psi)(1-\varepsilon))\lambda = \psi(1-\varepsilon)\lambda < \lambda$. The value of social losses are $\bar{J}^W(\lambda) = -(\varepsilon + (1-\psi)(1-\varepsilon))\lambda$ per unit of wealth. The recovered assets minus liabilities, $W + \bar{J}^W W$, are used to form new banks, that start with $z = 0$. Thus, the entry per unit of wealth loss is:

$$W^{new} = W + \bar{J}^W W.$$

Notice that $\bar{J}^W$ is the jump in aggregate wealth after the bank is liquidated whereas J^W are the private losses. As in the text, we think of the remaining value of the bank after liquidation, v_0, as some fixed share that the bank gets to keep after liquidations. The banker views this amount as returned to its shareholders and does not internalize the social losses. We impose a parametric restriction to guarantee that there is value left at the surviving banks. Thus:

$$v_0 \leq 1-(1-\psi(1-\varepsilon))\lambda \quad \forall\lambda \implies v_0 \leq 1-(1-\psi(1-\varepsilon))\kappa.$$

Equilibrium Rates. We assume a competitive environment. Given the linear technologies, the return on loans to banked firms is pinned down by their marginal product of capital:

$$r_t^L = A^L - \delta.$$

The user cost of capital of unbanked firms must equal the deposit rate:

$$r_t^D = A^d - \delta.$$

De facto, banks face a perfectly elastic supply of deposits and demand for loans. Implicitly, this assumes that unbanked firms are plenty and deposits are never scarce. By contrast, banks have limited wealth.

Aggregation. We now consider the heterogeneity across banks that occurs when there is delayed accounting. Let $g_t(z,W)$ denote the joint distribution of z and W. Then, aggregating across banks we obtain that total loans are:

$$\mathcal{L}_t = \int_0^\infty \int_0^\infty \lambda(z) W g_t(z,W)\, dzdW,$$

and likewise for deposits:

$$\mathcal{D}_t = \int_0^\infty \int_0^\infty (\lambda(z)-1) W g_t(z,W)\, dzdW.$$

The total equity at banks is:

$$\mathcal{W}_t = \int_0^\infty \int_0^\infty W g_t(z,W)\, dzdW.$$

Households hold wealth, N, in equity in non-banked firms and in the banking sector $\mathcal{W}$. The sum of their sectoral wealth adds to the total capital stock:

$$K_t = \mathcal{W}_t + N_t;$$

Operated capital differs from the allocation of equity. In particular, the capital stock is divided into:

$$K_t = K_t^L + K_t^D,$$

where K_t^D is the capital stock operated by households directly and K_t^L is the capital managed by banked firms, financed by borrowing loans. This latter capital equals the stock of bank loans because loans are used to finance the holdings of capital by banked firms: $K_t^L = \mathcal{L}_t$. Here, $\mathcal{L}_t$ is the aggregation of the individual loans L_t of a banks encountered earlier. Each bank chooses $D_t \geq 0$ standing for borrowed funds from non-banked firms. Thus, $\mathcal{D}_t = K_t^L - \mathcal{W}_t$, which is determined by the choice of leverage. Naturally, borrowed funds come from firms, $K_t^D + \mathcal{D}_t = N_t$.

Aggregate banks' value added is:

$$\Pi_t^B = r^L \mathcal{L}_t - r^D \mathcal{D}_t.$$

The value added of unbanked firms is:

$$\Pi_t^D = r^D D_t + A^D \left(N_t - D_t\right),$$

which includes deposits at banks and the remainder of operated capital. Banked firms' value added is:

$$\Pi_t^L = (A^L - r^L) K_t^L.$$

The unbanked firms and banks pay out exogenous dividend rates, $\bar{c}$ and c^D, respectively. Banked firms make zero profits in equilibrium. Hence, total consumption is:

$$\mathcal{C}_t = \bar{c}\mathcal{W}_t + c^D N_t. \tag{22}$$

The change in the aggregate equity of banks is:

$$\begin{aligned}
\dot{\mathcal{W}}_t &= \Pi_t^B - \bar{c}\mathcal{W}_t - \underbrace{\sigma\left(\omega_t + (1-\omega_t)\left(\varepsilon + (1-\psi)(1-\varepsilon)\right)\right)}_{\equiv \eta_t} K_t^L && (23) \\
&= (r^L - \eta_t)\mathcal{L}_t - r^D \mathcal{D}_t - \bar{c}\mathcal{W}_t, && (24)
\end{aligned}$$

where ω_t is the share of banks that suffer a default event and survive. Wealth in unbanked firms evolves as:

$$\dot{N}_t = \Pi_t^D - c^D N_t - \delta K_t^D.$$

Aggregate income is:

$$\begin{aligned}
\mathcal{Y}_t &= \Pi_t^B + \Pi_t^D + \Pi_t^L && (25) \\
&= A^L K_t^L + A^D K_t^D,
\end{aligned}$$

which sums to aggregate production. Capital investment is done by banked firms, as we show next.

Using (22) and (25), we have that:

$$\begin{aligned}
\mathcal{Y}_t - \mathcal{C}_t &= A^L K_t^L + A^D K_t^D - c^D N_t - \bar{c}\mathcal{W}_t \\
&= \left(r^L + \delta\right) K_t^L + A^D K_t^D - c^D N_t - \bar{c}\mathcal{W}_t \\
&= \left(r^L + \delta\right) K_t^L - r^D \mathcal{D}_t - \bar{c}\mathcal{W}_t + A^D K_t^D + r^D \mathcal{D}_t - c^D N_t \\
&= \dot{\mathcal{W}}_t + \dot{N}_t + \eta_t K_t^L + \delta K_t^L + \delta K_t^D \\
&= \dot{K}_t + \left(\eta_t + \delta\right) K_t^L + \delta K_t^D.
\end{aligned}$$

Thus, defining $\mathcal{I}_t$ as the change in capital plus total capital depreciated:

$$\dot{K}_t = \mathcal{I}_t - (\eta_t + \delta) K_t^L - \delta K_t^D.$$

Hence, we verify the income identity $\mathcal{I}_t \equiv \mathcal{Y}_t - \mathcal{C}_t$. We are now ready to introduce the welfare function.

C.2 Social Welfare Function

We work directly with the household's linear benefit from consumption as the notion of social welfare:

$$\mathcal{V}(N_0, \mathcal{W}_0) \equiv \mathbb{E}\left[\int_0^\infty \exp(-\rho t)\, \mathcal{C}_t dt | \mathcal{W}_0, N_0\right].$$

Immediate Accounting. Let's start with the immediate accounting case, assuming that all banks behave the same. We have that the social welfare function has an HJB representation:

$$\rho\mathcal{V}(N, \mathcal{W}) \equiv c^D N + \bar{c}\mathcal{W} + \mathcal{V}_N(N, \mathcal{W})\left(\Pi^D - \delta - c^D\right) N + \mathcal{V}_\mathcal{W}(N, \mathcal{W})\left(r^L\lambda\mathcal{W} - r^D(\lambda - 1)\mathcal{W} - \bar{c}\mathcal{W} - \eta\lambda\mathcal{W}\right).$$

This HJB is additive:

$$\mathcal{V}(N, \mathcal{W}) = \mathcal{F}(N) + \mathcal{P}(\mathcal{W})$$

where:

$$\rho\mathcal{F}(N) = c^D N + \mathcal{F}_N(N)\left(\Pi^d - \delta - c^D\right) N$$

and,

$$\rho\mathcal{P}(\mathcal{W}) = \bar{c}\mathcal{W} + \mathcal{P}_\mathcal{W}(\mathcal{W})\underbrace{\left(r^L\lambda - (\lambda - 1) r^D - \bar{c} - \eta\lambda\right)}_{\equiv \mu^\mathcal{W}}\mathcal{W}.$$

In turn, this value function is linear and thus:

$$\mathcal{P}(\mathcal{W}) = p\mathcal{W}$$

where p satisfies

$$\rho p = \bar{c} + p\mu^\mathcal{W},$$

and unpacking $\mu^\mathcal{W}$,

$$\mu^\mathcal{W} = r^L\lambda - (\lambda - 1) r^D - \bar{c} - \sigma\varepsilon\lambda \times \mathbb{I}_{[\lambda \le \min\{\Lambda, \Xi\}]} - \sigma(\varepsilon + (1 - \psi)(1 - \varepsilon))\lambda \times \mathbb{I}_{[\lambda \le \min\{\Lambda, \Xi\}]}.$$

Thus, we have the following result:

Problem 2 *[Planner's Optimal Leverage] Under immediate accounting, the maximization of social welfare requires the maximization of*

$$r^L\lambda - (\lambda - 1) r^D - \bar{c} - \sigma\varepsilon\lambda \times \mathbb{I}_{[\lambda \le \Lambda]} - \sigma(\varepsilon + (1 - \psi)(1 - \varepsilon))\lambda \times \mathbb{I}_{[\lambda > \Lambda]}.$$

Delayed Accounting. In sequential form, the objective function is:

$$\mathcal{V}(N_0, \mathcal{W}_0) \equiv \overbrace{\int_0^\infty \exp(-\rho t)\, c^D N_t dt}^{\mathcal{F}(N_0)} + \mathbb{E}\left[\int_0^\infty \int_0^\infty \int_0^\infty \exp(-\rho t)\, \bar{c} W_t g_t(z, W)\, dz dW dt\right].$$

We know that:

$$g_t(z, W) = \int_0^\infty \int_0^\infty G_t(z_t, W_t; z_0, W_0) \times g_0(z_0, W_0)\, dz_0 dW_0,$$

for some function $G_t(z_t, W_t; z_0, W_0)$ that yields a density of $\{z_t, W_t\}$ conditional on an initial condition $\{z_0, W_0\}$. Thus, the term in the objective is written as:

$$\int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \exp(-\rho t)\, \bar{c} W_t G_t(z_t, W_t; z_0, W_0)\, dz_t dW_t dt g_0(z_0, W_0)\, dz_0 dW_0$$

In turn, the term:

$$\int_0^\infty \int_0^\infty W_t G_t(z_t, W_t; z_0, \mathcal{W}_0)\, dz_0 dW_0 = \int_0^\infty \int_0^\infty \mathbb{E}[W_t | W_0 = W, z_0 = z]\, g_0(z_0, W_0)\, dz_0 dW_0.$$

Hence, the expected value of dividends by banks can be written as:

$$\mathcal{P}[\{g_0\}] \equiv \int_0^\infty \int_0^\infty P(z, W)\, g_0(z, W)\, dz dW.$$

where implicitly we have defined:

$$P(z, \mathcal{W}) = \mathbb{E}\left[\int_0^\infty \exp(-\rho t)\, \bar{c} W_t dt | \mathcal{W}_0 = \mathcal{W}, z_0 = z\right].$$

Next, we work with the Feynman-Kac formula to consider bank liquidations and the formation of new banks. In this case:

$$\rho P(z, W) = cW + P_W(z, W)\, \mu^W(z)\, W + P_z(z, W)\, \dot{z} + \sigma\left[P\left(z + \bar{J}^z, W + \bar{J}^W W\right) - P(z, W)\right].$$

and we have that:

$$\mu^W(z) \equiv r^L \lambda(z) - r^D(\lambda - 1) - \bar{c},$$

whereas the jump terms are:

$$\bar{J}^W = -\sigma \varepsilon \lambda \times \mathbb{I}_{\lambda(z) \leq \Lambda(z)} - \sigma(\varepsilon + (1 - \psi)(1 - \varepsilon))\, \lambda \times \mathbb{I}_{\lambda(z) > \Lambda(z)}$$

$$\bar{J}^z = J^z \times \mathbb{I}_{\lambda(z) \leq \Lambda(z)} - z \times \mathbb{I}_{[\lambda(z) > \Lambda(z)]}.$$

As before, we verify that $P(z, W)$ is scale independent. Thus:

Problem 3 *[Bank Optimization Problem] Under delayed accounting, the social value of an individual bank is given by $P(z, W) = p(z)\, W$ where:*

$$\rho p(z) = c + p(z)\, \mu^W(z) + p_z(z)\, \mu^z(z) + \sigma\left[p\left(z + \bar{J}^z\right)\left(1 + \bar{J}^W\right) - p(z)\right],$$

where

$$\bar{J}^W = -\sigma \varepsilon \lambda \times \mathbb{I}_{\lambda(z) \leq \Lambda(z)} - \sigma(\varepsilon + (1 - \psi)(1 - \varepsilon))\, \lambda \times \mathbb{I}_{\lambda(z) > \Lambda(z)}$$

and

$$\bar{J}^z = J^z \times \mathbb{I}_{\lambda(z) \leq \Lambda(z)} - z \times \mathbb{I}_{[\lambda(z) > \Lambda(z)]}.$$

D Additional Model Discussions

D.1 In Detail: Model Timing

Timing. To clarify the timing assumption, the top panels of Figure D.1 plot an example of a sample path of N_t and dN_t and the implied behavior of book leverage $\bar{\lambda}_t$ and Z_t. Assume that the bank arbirtrarilly decides to set book leverage to a constant, $\bar{\lambda}$. The Figure depicts a hypothetical scenario of a default event at time τ. At τ, the process N_t jumps following a path that is continuous from the right. This reflects in the discontinuous point in dN_t. The lower left panel depicts the path of $\bar{\lambda}$: the discontinuity represents the jump after the default event. In this example, $\bar{\lambda} + J^{\bar{\lambda}} < \Xi$, so the bank remains solvent. Critically, this happens because the bank cannot control its book leverage, via D, all the time. The lower right panel shows that Z_t also jumps the date of the shock. However, this variable is continuous from the left, which is why $\bar{\lambda}$ jumps for that instant. After the shock, the bank sells loans and transfers its losses into the stock of Zombie loans to return back to a constant book leverage path. The bank could have been liquidated if the jump lead to some point $\bar{\lambda} + J^{\bar{\lambda}} > \Xi$, even though the violation would have been for an infinitesimal period of time. This assumption is equivalent to the discrete time assumption that the shock occurs between periods. It can also be obtained as a limit process, where we have adjustment costs to selling loans that are taken to zero.

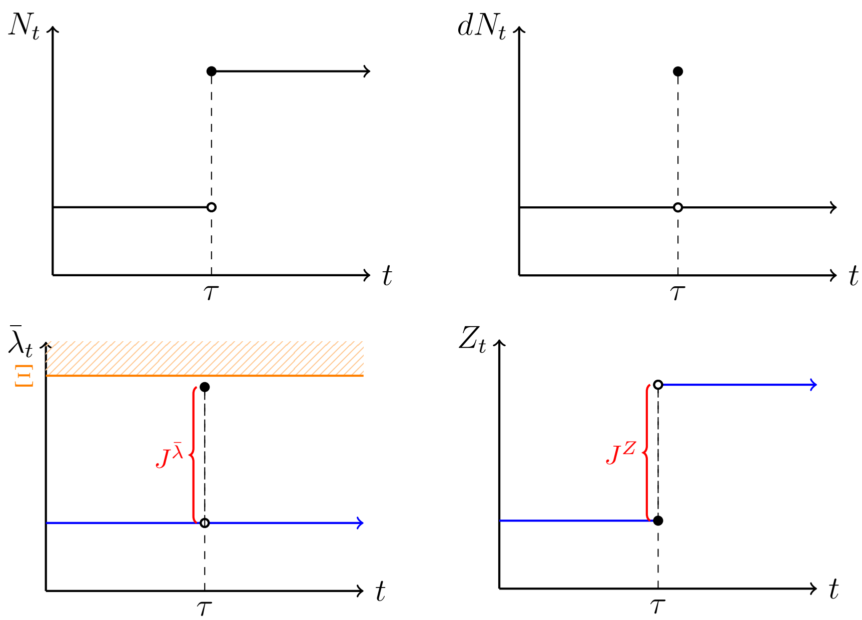


Figure D.1: Example: Timing Assumption

Notation and Definitions. We begin by presenting some definitions and deriving the laws of motion of the state variables of the model and other variables of interest. Recall that, for level variables, μ^x and J^x refer to the drift and jump components of the path of a variable x scaled by wealth W, respectively. For financial ratios, μ^x and J^x refer to the drift and jump components of the path of a variable x scaled without scaling.

Throughout the paper we use the following relationships that allow us to recover the original state variables $\left\{L, \bar{L}, D\right\}$ from the triplet $\{\lambda, z, W\}$:

$$
\begin{aligned}
L &= \lambda \cdot W && (26)\\
D &= (\lambda - 1) \cdot W && (27)\\
\bar{L} &= \lambda W + zW. && (28)\\
\bar{W} &= W + zW. && (29)
\end{aligned}
$$

We express the dividend-to-equity ratio as:

$$c \equiv C/W.$$

D.2 Variable Paths

Let τ be the time of the realization of a default event. Next, we describe the paths of variables in their canonical (integral) form.

Discontinuity Points at Default Events. We describe the behavior of each model variable at its discontinuity point.

Loans. We have that the controlled part of loans is:

$$\lim_{t \to \tau^-} L_t = \lim_{t \to \tau^-} \lambda_t W_t,$$

along a path where λ_t follows a continuous path prior to the default event.

Upon a default event, the fraction ε of loans is lost:

$$L_\tau = \lim_{t \to \tau^-} L_t - \varepsilon L_t.$$

Immediately after the jump, the bank sells loans at the amount S_τ. Therefore,

$$\lim_{t \to \tau^+} L_t = L_\tau - S_\tau.$$

Discrete loan events occur when there are no defaults, but z reaches z^o. The date τ of a loan sale event, we also have the same equation.

Deposits. Default events do not change the bank's liabilities. Thus,

$$\lim_{t \to \tau^-} D_t = D_\tau,$$

After a default event, the bank sells loans and reduces its stock of liabilities:

$$\lim_{t \to \tau^+} D_t = D_\tau - S_\tau.$$

Fundamental Equity. At the instant of the default event, equity and loans satisfy:

$$W_\tau = \lim_{t \to \tau^-} W_t - \varepsilon L_t$$

Fundamental equity remains unchanged with a loan sale:

$$\lim_{t \to \tau^+} W_t = W_\tau.$$

This limits always satisfy $W_t = L_t - D_t, \quad \forall t.$

Zombie Loans. As a result of a default event, zombie loans satisfy:

$$Z_\tau = \lim_{t\to\tau^-} Z_t = \lim_{t\to\tau^-} z_t W_t,$$

at the instant of the default event. The banker can choose to hide losses in the amount $H_\tau = \varepsilon \lim_{t\to\tau^-} \lambda_t W_t$. It optimally does so. Thus, zombie loans change with the hiding of losses immediately after the default event:

$$\lim_{t\to\tau^+} Z_t = Z_\tau + H_\tau.$$

Fundamental Leverage. For convenience, we define:

$$F_\tau \equiv \frac{S_\tau}{\lim_{t\to\tau^-} W_t}.$$

As a result of a default event, leverage is:

$$\lambda_\tau = \lim_{t\to\tau^-} \frac{L_t - \varepsilon L_t}{W_t - \varepsilon L_t} = (1-\varepsilon) \lim_{t\to\tau^-} \frac{\lambda_t}{1-\varepsilon\lambda_t},$$

at the instant of the default event. The jump in leverage corresponds to the uncontrolled part of leverage

$$\lambda_\tau - \lim_{t\to\tau^-} \lambda_t.$$

Considering the asset sales, fundamental leverage is follows:

$$\lim_{t\to\tau^+} \lambda_t = \frac{L_\tau + S_\tau}{W_\tau} = \frac{L_t - \varepsilon L_t + S_\tau}{W_t - \varepsilon L_t} = \lim_{t\to\tau^-} \frac{\lambda_t - \varepsilon\lambda_t}{1-\varepsilon\lambda_t} + \frac{1}{\lim_{t\to\tau^-} W_t} \frac{S_\tau}{(1-\varepsilon\lambda_t)}$$

Thus, leverage jumps to:

$$\lim_{t\to\tau^+} \lambda_t = \lim_{t\to\tau^-} \frac{\lambda_t - \varepsilon\lambda_t}{1-\varepsilon\lambda_t} + \frac{S_\tau}{(1-\varepsilon\lambda_t)},$$

considering the adjustment of loans.

Zombie Ratios. At the time of a default event, the zombie ratio satisfies:

$$z_\tau = \lim_{t\to\tau^-} \frac{Z_t}{W_t - \varepsilon L_t} = \lim_{t\to\tau^-} \frac{z_t}{1-\varepsilon\lambda_t},$$

at the instant of the default event. The zombie ratio remains unchanged with a loan sale:

$$\lim_{t\to\tau^+} z_t = z_\tau + \lim_{t\to\tau^-} \frac{\varepsilon\lambda_t}{1-\varepsilon\lambda_t}.$$

Book Loans. At the time of a default event, book loans satisfy:

$$\bar{L}_\tau = \lim_{t\to\tau^-} \bar{L}_t + \varepsilon\lambda_t W_t,$$

reflecting that the losses can be detected at the instant of the default event. However, immediately after the jump, the bank sells loans for the amount $-S_\tau$, and the sale is registered in the books.

Thus,

$$\lim_{t\to\tau^+} L_t = L_\tau - S_\tau + H_\tau.$$

Book Equity. Book equity jumps the instant of a default event. Thus:

$$\bar{W}_\tau = \lim_{t\to\tau^-} \bar{W}_t - \varepsilon\lambda_t W_t,$$

But it increases with the hiding of losses immediately after:

$$\lim_{t\to\tau^+} \bar{W}_t = \bar{W}_\tau + H_\tau.$$

Book Leverage. Book leverage remains jumps the instant of a default event. Thus:

$$\bar{\lambda}_\tau = \lim_{t\to\tau^-} \bar{\lambda}_t - \frac{\varepsilon\lambda_t}{1-\varepsilon\lambda_t},$$

but reverts back immediately after to:

$$\lim_{t\to\tau^+} \bar{\lambda}_t = \bar{\lambda}_\tau + \lim_{t\to\tau^-} \frac{\varepsilon\lambda_t}{1-\varepsilon\lambda_t} - \frac{S_\tau}{1-\varepsilon\lambda_t}$$

Little q. At the time of a default event, little q satisfies:

$$q_\tau = \lim_{t\to\tau^-} \frac{W_t - \varepsilon\lambda_t W_t}{\bar{W}_\tau - \varepsilon\lambda_t W_t}.$$

:

$$\lim_{t\to\tau^+} q_\tau = \lim_{t\to\tau^-} \frac{W_t - \varepsilon\lambda_t W_t}{\bar{W}_\tau},$$

reverts back with the jump in losses.

Discontinuity Points at Unforced Deleveraging Events. If at some date τ, the bank reaches some point in the stat space $\{z^o, W\}$ an decides to switch its leverage position discretely, i.e., if the optimal policy λ^* has a discontinuity pint, the paths of variables are as above, setting $\varepsilon = 0$ and

$$S_\tau = (1-\varepsilon\lambda)\left(\lim_{t\to\tau^+} \lambda_t - \lim_{t\to\tau^-} \lambda_t\right).$$

D.3 From Leverage to Loan Growth Ratios

While leverage is a control variable for banks, along a continuous path of the bank's state variables and controls, we can also describe growth rate of loans ι. Along a continuous path, the net investment in loans by a the bank is:

$$\iota \equiv I/L - \delta,$$

where δ can represent the repayment share of past loans and I a flow of new loans. The flow of new loans and, thus, ι should be consistent with the bank's leverage decision.

We note the relationship between the growth in loans and the state variable z, along a continuous path for loans.

$$L = \lambda W \Rightarrow \mu^L W = \dot{\lambda} W + \lambda \mu^W W \Rightarrow \dot{\lambda} = \mu^L - \lambda\mu^W.$$

Replacing the values for these variables, we obtain:

$$\dot{\lambda} = \iota\lambda - \lambda\mu^{W} \Rightarrow \iota = \frac{\dot{\lambda}}{\lambda} + \mu^{W}.$$

Thus, the growth rate of loans is the sum of growth of leverage plus the growth in equity.

Consistency. Along a continuous path, if the solution to the bank's problem is given by some optimal leverage decision, $\lambda^{*}(z)$. In this case:

$$\dot{\lambda} = \lambda_{z}^{*}(z)\,\dot{z}.$$

Thereby, we obtain that the growth in loans is given by:

$$\iota = \frac{\lambda_{z}^{*}(z)}{\lambda^{*}(z)}\dot{z} + \mu^{W}(z).$$

Likewise, upon a default event:

$$\left(\lambda + J^{\lambda} + \bar{J}^{\lambda}\right)\left(W + J^{\lambda}\right) - \lambda W = SdN_{t}.$$

D.4 Derivation of Laws of Motion from Discrete-Time Analogs

Summary Table. The following table summarizes the drift and jump terms of different variables as functions of $\{\lambda, z, W\}$.

Variable	Drift	Jumps	Sale
State Variables and Financial Ratios			
W	$\left[r^{L}\lambda - r^{D}(\lambda-1) - c\right]W$	$-\varepsilon\lambda W$	0
Z	$-\alpha Z$	$\varepsilon\lambda W$	0
z	$-z\left(\alpha + \mu^{W}\right)$	$\varepsilon\lambda\left(\frac{z+1}{1-\varepsilon\lambda}\right)$	0
λ	$\iota\lambda - \lambda\mu^{W}$	$\frac{\varepsilon\lambda}{1-\varepsilon\lambda}(\lambda-1)$	$F(z)/(1-\varepsilon\lambda)$
Deduced Variables			
L	$\iota\lambda W$	$-\varepsilon\lambda W$	$F(z)$
D	$\left[r^{D}(\lambda-1) - r^{L}\lambda + \iota\lambda + c\right]W$	0	$F(z)$
$\bar{L}$	$(\iota\lambda - \alpha z)W$	0	$F(z)$
$\bar{W}$	$\left(r^{L}\lambda - r^{D}(\lambda-1) - \alpha z\right)W$	0	0
q	$\frac{\alpha z}{(1+z)^{2}}$	$-\varepsilon\lambda q$	0
$\bar{\lambda}$	$\frac{1}{1+z}\left(\iota\lambda - \lambda\mu^{W} - \alpha z\frac{1-\lambda}{\lambda+z}\right)$	0	$F(z)/q$
s	$-s'(z)\left(\alpha + \mu^{W}\right)z$	$s\left(z + \varepsilon\lambda\left(\frac{z+1}{1-\varepsilon\lambda}\right)\right) - s(z)$	0

Table 7: Drifts and Jumps of Variables

We present some observations that derive these laws of motion and provided preliminary results that aid the proof of the main propositions in the paper.

Preliminary Results 1: derivations of laws of motion. Here, we provide an explicit derivation of the law of motion of bank equity, starting from a discrete time formulation. In a discrete time formulation, with time interval Δ, the bank receives a default shock $\varepsilon < 1$ with probability $\sigma\Delta$. Let:

$$N_{t+\Delta} - N_t = \begin{cases} 0 & \text{with prob } 1 - \sigma\Delta \\ 1 & \text{with prob } \sigma\Delta \end{cases}$$

denote a default event process. Recall that dN is a Poisson process.

Loans. Now consider a time interval of length Δ. The law of motion for fundamental loans satisfies:

$$L_{t+\Delta} = (1 - \delta\Delta) L_t + I_t\Delta - \varepsilon L_t (N_{t+\Delta} - N_t) + F_t,$$

with the interpretation that the first term is the non-maturing fraction of loans, the second are loan issuances, and the third are losses in a time interval. Subtracting L_t from both sides and taking $\Delta \to 0$, we obtain the following law of motion:

$$dL = (I - \delta L) dt - \varepsilon L dN + F^L.$$

The interpretation of dL is important. As all differentials, it is a stand in for notation. We should interpret as a limit of a rate of change as $\Delta \to 0$; likewise, dN as the limit Poisson event corresponding to $N_{t+\Delta} - N_t$, as $\Delta \to 0$ and F^L as the process of asset sales.[59] The key is that this differentials must represent the paths in D.2.

We express this law of motion in terms of net-worth, replacing (26), to obtain:

$$dL = \iota\lambda W dt - \varepsilon\lambda W dN + F^L. \tag{30}$$

Consistent with our notation, we define the drift and jumps relative to wealth are given by:

$$\mu^L \equiv \iota\lambda \text{ and } J^L \equiv -\varepsilon\lambda.$$

Deposits. For deposits we have that:

$$D_{t+\Delta} = \left(1 + r^D\Delta\right) D_t - \left(r^L\Delta + \delta\Delta\right) L_t + I_t\Delta + C_t\Delta + F_t$$

with the interpretation that the first term is the increase in deposits that results from paying interest with deposits; the second term is the reduction in deposits by the interest and principal payments on outstanding loans; the third term is the increase in deposits as a result of loan issuances; and the final term is dividend payments, all paid with deposits. Taking $\Delta \to 0$, again we obtain the following law of motion:

$$dD = \left[r^D D - \left(r^L + \delta\right) L + I + C\right] dt + F_t.$$

We express this law of motion in terms of wealth, by using (29), to obtain:

$$dD = \left[r^D (\lambda - 1) - r^L\lambda + \iota\lambda + c\right] W dt + F_t. \tag{31}$$

[59] Likewise, we may wish to consider the path of loans conditional on no Poisson events, as wells as the expected path of loans, respectively:

$$\dot{L} = (I - \delta L) \quad \dot{L} = (I - \delta L) - \varepsilon L\sigma + F.$$

We define the growth rate of deposits relative to net-worth:

$$\mu^D \equiv r^D (\lambda - 1) - r^L \lambda + \iota\lambda + c \text{ and } J^D = 0.$$

Fundamental Equity. Next, we present the evolution of fundamental equity:

$$\begin{aligned} dW &= dL - dD \\ &= \left[\mu^L - \mu^D\right] W dt + J^L dN \qquad (32) \\ &= \left[\underbrace{r^L \lambda - r^D (\lambda - 1)}_{\text{levered returns}} - \underbrace{c}_{\text{dividend rate}}\right] W dt - \underbrace{\varepsilon \cdot \lambda}_{\text{loss rate}} W dN. \qquad (33) \end{aligned}$$

where the second line uses the laws of motion in (30) and (31). The interpretation of this expression is natural: the terms multiplying rates represent the net interest margin on the bank, which are the banks levered return; the second term are the capital gains that are accounted immediately as the bank creates an asset that can be worth more or less than a liability; the third term is the banks' dividend rate; and the final term is the loss rate, which scales with leverage.

Define the drift of the growth rate of bank equity as:

$$\mu^W \equiv r^L \lambda - r^D (\lambda - 1) - c$$

and denote the jump component of wealth as:

$$J^W \equiv -\varepsilon\lambda W = J^L W.$$

This verifies the following result.

Lemma 2 *If λ is independent of W, the model satisfies growth independence.*

Importantly, we note that the bank accumulates wealth over time, but does not make accounting profits by issuing deposits.

Zombie loans. The law of motion for zombie loans satisfies:

$$dZ = -\alpha\Delta Z - \varepsilon L_t (N_{t+\Delta} - N_t)$$

Taking $\Delta \to 0$, we obtain the following law of motion:

$$dZ = -\alpha Z dt + \varepsilon L dN = -\alpha z W dt + \varepsilon\lambda W dN. \qquad (34)$$

Thus,

$$\mu^Z \equiv -\alpha Z \textbf{ and } J^Z \equiv -\varepsilon\lambda W = J^L W.$$

Book Loans. The law of motion for book loans satisfies:

$$\bar{L}_{t+\Delta} = (1 - \delta\Delta) L_t + I_t\Delta - \alpha\Delta \left(\bar{L}_t - L_t\right) - \varepsilon L_t (N_{t+\Delta} - N_t) + F_t,$$

with the interpretation that the first term represents how book loans fall as the principal of fundamental loans gets repaid; the second term increases book loans by newly issued loans; the third term decreases book loans at the speed of loan loss recognition α times the gap in the book versus fundamental loans; and the final term is the fraction of losses recognized in books upon receiving a default shock. Taking $\Delta \to 0$, we obtain the following law of motion:

$$d\bar{L} = (-\delta L + I)\, dt - \alpha \left(\bar{L} - L\right) dt + F_t.$$

We express this law of motion, by using (28), in terms of wealth to obtain:

$$d\bar{L} = [\iota\lambda - \alpha z]\, W dt + F_t. \tag{35}$$

We define the growth rate of book loans and the jump relative to net-worth accordingly:

$$\mu^{\bar{L}} \equiv \iota\lambda - \alpha z.$$

Book Equity. Book equity is the difference between book loans and deposits:

$$\bar{W}_t = \bar{L}_t - D_t.$$

Thus, the differential is:

$$d\bar{W}_t = \left(r^L\lambda - r^D(\lambda - 1) - c - \alpha z\right) W dt,$$

whereby

$$\mu^{\bar{W}} \equiv \left(r^L\lambda - r^D(\lambda - 1) - c - \alpha z\right).$$

Law of motion of zombie ratio. Employing the formula for the differential of a ratio we get:

$$\begin{aligned} \mu^z &\equiv z\left(\frac{\mu^Z W}{Z} - \frac{\mu^W W}{W}\right) \\ &= z\left(\frac{-\alpha z W}{Z} - \frac{\mu^W W}{W}\right) \\ &= -z\left(\alpha + \mu^W\right). \end{aligned} \tag{36}$$

Next, we derive the two possible jumps for z.

Upon a default event, we have that:

$$J^z \equiv \frac{Z + \varepsilon L}{W - \varepsilon L} - z = \frac{z + \varepsilon\lambda}{1 - \varepsilon\lambda} - z = \varepsilon\lambda\left(\frac{z+1}{1-\varepsilon\lambda}\right) = -J^W\left(\frac{z+1}{1-\varepsilon\lambda}\right).$$

Law of motion for q. Next, we produce the law of motion for leverage q. Recall that

$$q = \frac{W}{\bar{W}} = \frac{W}{W+Z} = \frac{1}{1+z}.$$

Thus, the continuous portion of q satisfies:

$$\mu^q \equiv \frac{1}{1+z} \cdot \frac{-\mu^z}{1+z} = \frac{\alpha z}{(1+z)^2}. \tag{37}$$

The jump upon an unrecognized default event is:

$$J^q \equiv \frac{W - \varepsilon L}{W - \varepsilon L + Z + \varepsilon L} - q = \frac{1 - \varepsilon\lambda}{1+z} - \frac{1}{1+z} = -\varepsilon\lambda q = J^w q.$$

Leverage. Next, we derive the law of motion for leverage λ given a choice of ι and c, along the continuous path of bank's variables. Employing the formula for the differential of a ratio we get:

$$\begin{aligned} \mu^{\lambda} &= \lambda\left(\frac{\mu^{L}W}{L}-\frac{\mu^{W}W}{W}\right) \\ &= \lambda\left(\frac{\iota\lambda W}{L}-\frac{\mu^{W}W}{W}\right) \\ &= \lambda\left(\iota-\mu^{W}\right). \end{aligned} \tag{38}$$

Upon a default shock, the discontinuous jump in leverage is given by:

$$J^{\lambda}=\frac{L-\varepsilon\lambda W}{W-\varepsilon\lambda W}-\frac{L}{W}=\left(\frac{(1-\varepsilon)\cdot\lambda}{1-\varepsilon\lambda}-\lambda\right)=\varepsilon\lambda\cdot\frac{\lambda-1}{1-\varepsilon\lambda}.$$

Therefore, combining the drift and jump portions of the law of motion, we obtain:

$$d\lambda=\left(\iota-\mu^{W}\right)\lambda dt-\varepsilon\lambda\cdot\frac{\lambda-1}{1-\varepsilon\lambda}dN. \tag{39}$$

The interpretation of this law of motion is that leverage increases with the issuance rate, falls as loans mature and falls as the bank makes earns income on its current portfolio, μ^{W}. We thus have:

$$\mu^{\lambda}=\left(\iota-\mu^{W}\right)\lambda,$$

and for the jump term, we obtain

$$J^{\lambda}=\varepsilon\lambda\cdot\frac{\lambda-1}{1-\varepsilon\lambda}=-J^{W}\frac{\lambda-1}{1-\varepsilon\lambda}.$$

Naturally, leverage jumps with defaults, and more so the more levered the bank is.

Next, immediately after a jump in leverage, the bank sells a number of loans F.

Book Leverage. Next, we produce the law of motion for book leverage $\bar{\lambda}$. Recall that

$$\bar{\lambda}=\frac{\bar{L}}{\bar{W}}=\frac{L+Z}{W+Z}=\frac{\lambda+z}{1+z}.$$

Thus, the continuous portion of $\bar{\lambda}$ satisfies:

$$\mu^{\bar{\lambda}}\equiv\bar{\lambda}\left(\frac{\mu^{\lambda}}{\lambda+z}+\mu^{z}\left(\frac{1}{\lambda+z}-\frac{1}{1+z}\right)\right)=\frac{1}{1+z}\left(\mu^{\lambda}+\mu^{z}\frac{1-\lambda}{\lambda+z}\right). \tag{40}$$

The jump upon an unrecognized default event is:

$$J^{\bar{\lambda}}\equiv\frac{L-\varepsilon L+Z+\varepsilon L}{W-\varepsilon L+Z+\varepsilon L}-\bar{\lambda}=\frac{\lambda+z}{1+z}-\bar{\lambda}=0.$$

D.5 In Detail: Solution Under Immediate Accounting

Recall Proposition 1 in the body of the text. Figure D.2 depicts the objective function $\Omega(\lambda)$ in (11), as a function of λ. The figure illustrates the trade-off between levered returns and liquidation risk. Notice that $\Omega(\lambda)$ displays three segments: (I) In the *liquidation region* (i.e., $\lambda > \kappa$) the bank is immediately liquidated—the value of $\Omega(\lambda)$ is zero. (II) In the *solvency region,* leverage is lower than the shadow-boundary leverage, (i.e., $\lambda \in [1, \Lambda]$). The bank effectively circumvents liquidation risk, as its leverage remains below the liquidation threshold, even in the event of a default. (III) In the *liquidation-risk region* (i.e., $\lambda \in (\Lambda, \kappa]$), the bank is liquidated once a loan default event occurs.

Panel (a) shows $\Omega(\lambda)$ maximized at the shadow boundary, while Panel (b) depicts its maximization at the liquidation boundary. The same pattern is true with delayed accounting.

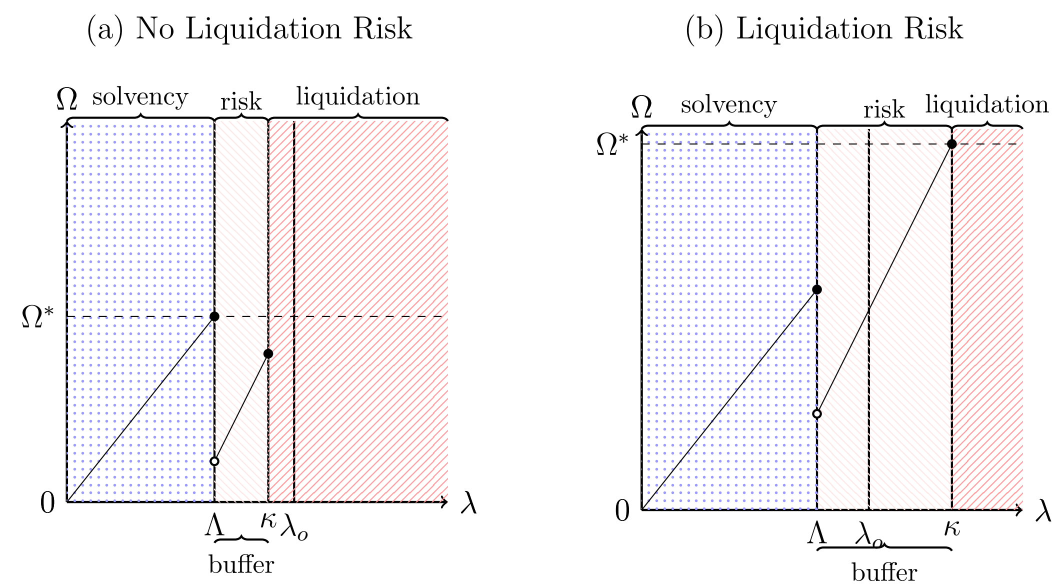


Figure D.2: Return vs. Liquidation Risk Tradeoff

Notes: The two panels plot the value of the bank's objective for different values of λ for the case with immediate loan loss recognition for different values of the fundamental leverage constraint, κ—under $\kappa < \Xi$. In the left panel the bank prefers to set leverage at the shadow boundary and not risk liquidation. In the right panel, the bank risks liquidation.

In the solvency region, both the levered return, expressed as $\left(r^L - r^D\right)\lambda$, and the default losses, denoted as $-\sigma\varepsilon\lambda$, are proportional to leverage. According to Assumption 1, the expected levered return is positive, leading to an increase in the objective function within this segment. In contrast, in the liquidation risk region, while the levered return $\left(r^L - r^D\right)\lambda$ continues to be linearly related to leverage, loan losses no longer influence the bank's objective, resulting in an increased slope of the objective function. However, the fixed expected liquidation cost, represented by $\sigma\left(\frac{v^o}{v^*} - 1\right)$, is deducted from the objective function. When leverage transitions from the solvency region to the liquidation risk region, there is a discontinuous drop in the objective, reflecting the bank's anticipated liquidation cost. Consequently, $\Omega(\lambda)$ exhibits two local maxima: one at the shadow boundary, Λ, and another at the liquidation boundary, κ. Therefore, the leverage that maximizes expected returns, Ω^*, is situated either at the shadow boundary or at the liquidation boundary

$$\Omega^* = r^D + \max\left\{\overbrace{\left(r^L - r^D\right)\kappa - \sigma\left(1 - \frac{v^o}{v^*}\right)}^{\text{liquidation boundary}}, \quad \overbrace{\Lambda\left(\left(r^L - r^D\right) - \varepsilon\sigma\right)}^{\text{shadow boundary}}\right\},$$

as noted in the body of the text. Figure D.2 illustrates the two scenarios: Panel (a) shows $\Omega(\lambda)$ maximized at the shadow boundary, while Panel (b) depicts its maximization at the liquidation boundary. The same pattern is true with delayed accounting. Corollary 1 dictates parametric conditions under which one case is the optimal solution.

D.6 In Detail: Market-based Liquidations and Insolvency

Next, we explain the connection between insolvency and liquidations. The bank is solvent after a default shock if:

$$(L - \varepsilon L) - D \geq 0 \rightarrow (1 - \varepsilon)\lambda - (\lambda - 1) \geq 0.$$

Or, re-arranging:

$$\lambda \leq 1/\varepsilon.$$

The bank satisfies the market-based constraint and, thus, avoids liquidation after the shock if:

$$\lambda \leq \frac{1}{\kappa^{-1} + \varepsilon\left(1 - \kappa^{-1}\right)} \equiv \Lambda < \kappa.$$

If $\kappa \rightarrow \infty$, then, the condition es equivalent to a solvency condition, $\lambda \leq 1/\varepsilon$

We have the following relations:

- if $\kappa \rightarrow \infty$, the shadow boundary is equivalent to a solvency condition after a shock
- for finite values of κ, the bank is solvent if banks stay at the shadow boundary
- if κ is set below $1/\varepsilon$, the bank may be liquidated while always being solvent
- if κ is set above $1/\varepsilon$, and λ is chosen above $1/\varepsilon$, the bank is insolvent after the first shock
- if $\kappa = 1/\varepsilon$, the shadow boundary is 1. In that case, the bank is always liquidated after the first shock if it levered.

D.7 In Detail: Epstein-Zin Preferences

In the quantitative section, we use Duffie-Epstein preferences. Under these preferences:

$$V_t = \mathbb{E}_t \left[\int_t^{\infty} f\left(C_s, V_s\right) ds \right],$$

where the aggregator f is given by:

$$\begin{aligned} f\left(C, V\right) &\equiv \frac{\rho}{1-\theta} \left[\frac{C^{1-\theta} - \left\{ \left(1-\psi\right) V + \psi \right\}^{\frac{1-\theta}{1-\psi}}}{\left\{ \left(1-\psi\right) V + \psi \right\}^{\frac{1-\theta}{1-\psi}-1}} \right] \\ &= \frac{\rho}{1-\theta} \left\{ \left(1-\psi\right) V + \psi \right\} \left[\frac{C^{1-\theta}}{\left\{ \left(1-\psi\right) V + \psi \right\}^{\frac{1-\theta}{1-\psi}}} - 1 \right]. \end{aligned}$$

In this representation, V_t stands for flow utility. We have some limits of interest.

CRRA Case. Consider the following limit:

$$\lim_{\psi \to \theta} \frac{\rho}{1-\theta} \left[\frac{C^{1-\theta} - \left\{ \left(1-\psi\right) V + \psi \right\}^{\frac{1-\theta}{1-\psi}}}{\left\{ \left(1-\psi\right) V + \psi \right\}^{\frac{1-\theta}{1-\psi}-1}} \right] = \rho \frac{C^{1-\theta} - \theta}{1-\theta} - \rho V.$$

Thus, the HJB equation (56) becomes:

$$\rho V = \rho \frac{C^{1-\theta} - \theta}{1-\theta} + V_W \mu^W + V_Z \mu^Z + \theta J^V.$$

This formulation is consistent with standard time-separable utility:

$$V_t = \rho \mathbb{E}_t \left[\int_t^{\infty} \exp\left(-\rho s\right) \frac{C^{1-\theta} - \theta}{1-\theta} ds \right],$$

which is the standard representation of utility expressed in flows.

Smoothed Dividends and Risk Neutrality Case. First, we present the limit as risk-aversion vanishes:

$$\lim_{\psi \to 0} f\left(C, V\right) = \frac{\rho}{1-\theta} V \left[\frac{C^{1-\theta}}{V^{1-\theta}} - 1 \right].$$

and for the derivative with respect to dividends we obtain:

$$\lim_{\psi \to 0} f_c\left(C, V\right) = \rho C^{-\theta} V^{\theta}.$$

Thus, the HJB equation in this case is:

$$0 = f\left(C_s, V_s\right) + V_W \mu^W + V_Z \mu^Z + \theta J^V.$$

We use this representation in the quantitative analysis.

Baseline case. Consider the further limit:

$$\lim_{\psi, \theta \to 0} f\left(C, V\right) = \rho C - \rho V. \tag{41}$$

In the baseline case of the model we assume $C = cW$ is given by a constant dividend policy. Then, equation (56) becomes:

$$\rho V = \rho c W + V_W \mu^W + V_Z \mu^Z + \theta J^V,$$

which is a scalar transformation of the one in the main text.

D.8 Microfoundation: Regulatory Liquidations

In the body of the paper we argue that whereas regulators cannot directly observe the amount of zombie loans held by banks, they could infer these through market prices. Moreover, we argue that during the instants where banks suffer defaults, regulators have a window of opportunity to intervene and liquidate them. Even though regulators do not observe bank accounting books in real time, market values, which would anticipate a successful intervention, could reveal the violation of regulatory constraints. In turn, if the intervention did not happen, banks would continue operating and immediately hide losses, only to reveal them later. This explains why even if regulators have the same information, they could only liquidate banks when default events make bank leverage cross the liquidation boundary. Here, we present a sequential form game, based on ideas on the costly-state verification model of Townsend (1979), which provides a micro-foundation for these features of the model.

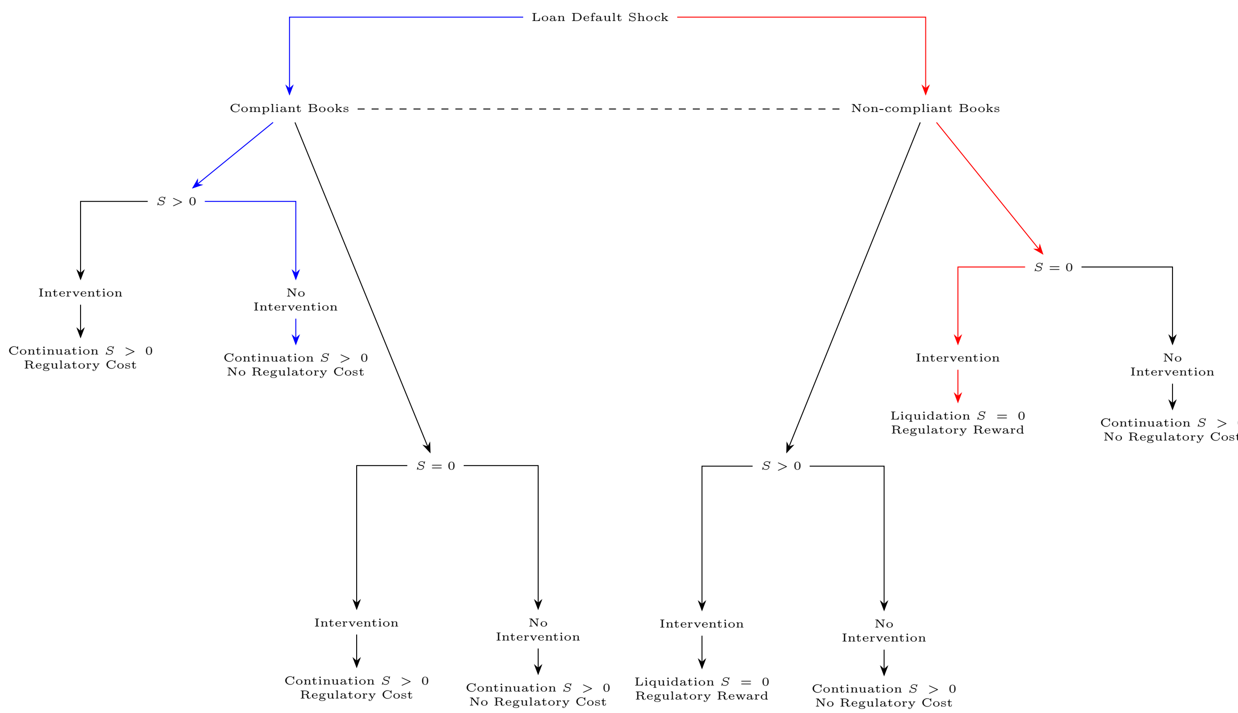


Figure D.3: Microfoundation for Regulatory Liquidations

The extensive form of this game is depicted in Figure D.3. Consider an instant of time t which unfolds starting with a loan default shock, determining the bank's condition as either maintaining regulatory compliant books or non-compliant books, considering current losses, but not past losses which are already hidden. The state of solvency is unknown to the regulator, but known to investors. The initial event sets the sequences of decisions by investors and the regulator, leading to distinct outcomes.

Following the shock, investors evaluate the bank's equity value, reflecting rational expectations regarding the continuation of the bank:

- $S > 0$: if investors assign a positive value to the bank's equity, anticipating the regulator's actions and the outcome of those actions.

- $S = 0$: Investors consider the bank's equity valueless, anticipating the regulators actions and the outcome of those actions.

Then, the regulator decides to intervene, conditioning on whether the price is positive or not:

- Intervention: the bank is intervened and, based on its books, the bank continues to operate only if it is solvent in its books.
- No Intervention: the regulator decides against taking any action, and the bank continues to operate regardless of its state.

The game's outcomes and payoff depend on the state of the bank and the actions of the regulator.

- Intervention of a compliant bank: The regulator incurs a cost for intervening. Investors do not lose anything.
- Intervention of an non-compliant bank: The bank is liquidated and the regulator receives a reward for taking timely action to mitigate further financial system risks. Investors get a value of zero and the price jumps to zero.
- No Intervention: the regulator does not gain or lose anything. The value of the bank remains positive.

We consider sub-game perfect equilibria of this game. The color-coded path represents one sub-game perfect equilibrium within the context of the game. Specifically, the path depicted in blue is a sub-game perfect equilibrium consistent with rational prices. If the bank is solvent the price is positive, signaling that a regulatory intervention will only lead to a cost to the regulator. The continuity of the bank guarantees that a positive price is indeed rational.

Likewise, the path colored in red is also a sub-game perfect equilibrium, delineating the optimal course of action in the event the bank is revealed to have insolvent books by low prices. In anticipation of a regulatory intervention, the price jumps to zero, thereby revealing to regulators the possibility of a successful intervention.

We note that another sub-game perfect equilibrium occurs when the price is always positive, revealing no information, and leaving the regulator oblivious. We do not consider this case, because, as we noted, it is inconsistent with a regulatory buffer.

E Proofs and Derivations

E.1 Proof of Lemma 1

This section derives the liquidation and shadow boundaries.

Notation. To simplify notation, let τ define the instant of a jump in X_t. We use:

$$X_{\tau^-} = \lim_{t \to \tau^-} X_t \quad \text{and} \quad X_{\tau^+} = \lim_{t \to \tau^+} X_t,$$

and X_τ is the value of the variable at the moment of the jump.

Liquidation Boundary. The regulatory constraint is

$$\bar{L}_t \leq \Xi \cdot \bar{W}_t \Leftrightarrow \bar{\lambda}_t \leq \Xi \quad , \forall t, \tag{42}$$

as we noted in the main body of the text. We express the regulatory capital requirement in terms of the pair $\{\lambda, z\}$. Recall that:

$$\bar{\lambda} \equiv \frac{\bar{L}}{\bar{W}} = \frac{\lambda W + Z}{W + Z} = \frac{\lambda + z}{1 + z}. \tag{43}$$

Combining (43) with (42), we obtain:

$$\frac{\lambda_t + z_t}{1 + z_t} \leq \Xi \Rightarrow \lambda_t \leq \Xi + (\Xi - 1)\, z_t. \tag{44}$$

Next, consider the market-based constraint:

$$\lambda_t \leq \kappa. \tag{45}$$

Thus, combining (44) and (45), we obtain:

$$\lambda \leq \Gamma(z) = \min\{\kappa, \Xi + (\Xi - 1)\, z\}. \tag{46}$$

Hence, the liquidation boundary can be written independent of W. The market based constraint is tighter than the regulatory constraint whenever:

$$\kappa \leq \Xi + (\Xi - 1)\, z \rightarrow z \geq \frac{\kappa - \Xi}{\Xi - 1}.$$

Thus, for any

$$z_t \geq z^\ell \equiv \frac{\kappa - \Xi}{\Xi - 1},$$

the market based constraint is tighter.

Shadow Boundary. The regulatory constraint can be expressed in terms of the capital buffer:

$$X_t \equiv \Xi \cdot \bar{W}_t - \bar{L}_t\ .$$

The bank satisfies the constraint whenever:

$$X_t \geq 0, \quad \forall t.$$

Let τ be be the instant of a default. Assume that the regulator could verify the fraction β of losses on impact, i.e., it observes the losses $\beta\varepsilon L_{\tau}$.

- When $\beta = 1$, we are in the main case studied in the paper. In this case, the bank can hide losses immediately after. Zombie loans increase immediately after τ.

- When $\beta = 0$, we are in alternative case where regulators never observe losses. Zombie loans increase immediately at τ.

The capital buffer at τ is:

$$\begin{aligned} X_{\tau} &= \Xi\left(\bar{W}_{\tau} - \bar{L}_{\tau}\right) \\ &= \Xi\left(\bar{W}_{\tau-} + \beta\varepsilon L_{\tau-} - \left(\bar{L}_{\tau-} + \beta\varepsilon L_{\tau-}\right)\right). \end{aligned}$$

Next, we write the capital buffer at τ in terms of fundamental variables:

$$\begin{aligned} \frac{X_{\tau}}{W_{\tau}} &= \Xi \cdot (1 + z_{\tau-} + \beta\varepsilon\lambda_{\tau-}) - (\lambda_{\tau-} + z_{\tau-} + \beta\varepsilon\lambda_{\tau-}) \rightarrow \\ &= \Xi + (\Xi - 1)\, z_{\tau-} - \lambda_{\tau-} - (\Xi - 1)\,\beta\varepsilon\lambda_{\tau-}. \end{aligned}$$

The bank can guarantee solvency at τ if $X_{\tau}/W_{\tau} \geq 0$. Hence, for any t, the bank avoids liquidation after a shock as long as:

$$0 \leq \Xi + (\Xi - 1)\, z_{\tau-} - (1 + (\Xi - 1)\,\beta\varepsilon)\,\lambda_{\tau-}.$$

Re-arranging, this implies that the bank remains solvent if

$$\lambda_{\tau-} \leq \frac{\Xi + (\Xi - 1)\, z_{\tau-}}{1 + (\Xi - 1)\,\beta\varepsilon}. \tag{47}$$

Setting $\beta = 1$ delivers the condition in the body of the paper. Next, we derive the shadow boundary, for $\beta \in \{0, 1\}$.

Shadow Boundary in Case $\beta = 1$. The bank satisfies the market-based constraint at τ if:

$$\lambda_{\tau-} + J^{\lambda}(\lambda_{\tau-}) = \lambda_{\tau-}\frac{(1-\varepsilon)}{1 - \varepsilon\lambda_{\tau-}} < \kappa.$$

Thus, the is survives the shock if:

$$\lambda_{\tau-} \leq \frac{\kappa}{1 - \varepsilon + \varepsilon\kappa}. \tag{48}$$

Combining (47), for $\beta = 1$, and (48), we obtain the shadow boundary; the bank survives a default event as long as

$$\lambda \leq \Lambda(z) \equiv \min\left\{\frac{\kappa}{1 - \varepsilon + \varepsilon\kappa}, \frac{\Xi + (\Xi - 1)\, z}{1 - \varepsilon + \Xi\varepsilon}\right\},$$

for z being the zombie ratio at the instant of a default.

In fact, the shadow boundary can be written as the pair $\{z, \lambda\}$ satisfying:

$$\lambda + J^{\lambda}(\lambda) = \min\left\{\kappa, \Xi + (\Xi - 1)\, J^{z}(z, \lambda)\right\}.$$

Recall that the value of $J^{z}(z, \lambda) = z\frac{1}{1-\varepsilon\lambda}$ and the value of $\lambda + J^{\lambda}(\lambda) = \lambda\frac{(1-\varepsilon)}{1-\varepsilon\lambda}$. For values of

$\kappa = \Xi + (\Xi - 1) J^z(z, \lambda)$, we have that:

$$\lambda \frac{(1-\varepsilon)}{1-\varepsilon\lambda} = \Xi + (\Xi - 1)\frac{z}{1-\varepsilon\lambda} \to \lambda = \frac{\Xi + (\Xi - 1) z}{1 + (\Xi - 1)\varepsilon},$$

thus verifying the case where the bank avoids liquidation under the regulatory constraint with $\beta = 1$.

Next, we compute the values of z such that the market-based liquidation boundary is smaller than the regulatory counterpart. This value is

$$z^s \equiv \frac{1-\varepsilon}{1-\varepsilon+\varepsilon\kappa} \times \frac{\kappa - \Xi}{\Xi - 1} = \frac{1-\varepsilon}{1-\varepsilon+\varepsilon\kappa} \times z^m.$$

and satisfies:

$$\frac{\Xi + (\Xi - 1) z^s}{1-\varepsilon+\varepsilon\Xi} = \frac{\kappa}{1-\varepsilon+\varepsilon\kappa}.$$

Hence, we have that the shadow boundary is:

$$\Lambda(z) = \begin{cases} \frac{\kappa}{1-\varepsilon+\varepsilon\kappa} & \text{if } z > z^s \\ \frac{\Xi+(\Xi-1)z}{1-\varepsilon+\varepsilon\Xi} & \text{if } z \le z^s. \end{cases}$$

We verify the continuity of the shadow boundary at z^s:

$$\Lambda(z^s) = \frac{\Xi + (\Xi-1) z^s}{1-\varepsilon+\Xi\varepsilon} = \frac{\Xi + \frac{\kappa-\Xi}{1-\varepsilon+\varepsilon\kappa}}{1-\varepsilon+\Xi\varepsilon} = \frac{\Xi + \frac{\kappa-\Xi-\varepsilon(\kappa-\Xi)}{1-\varepsilon+\varepsilon\kappa}}{1-\varepsilon+\Xi\varepsilon} = \frac{\frac{\Xi(\varepsilon\kappa)+\kappa-\varepsilon\kappa}{1-\varepsilon+\varepsilon\kappa}}{1-\varepsilon+\Xi\varepsilon} = \frac{\kappa}{1-\varepsilon+\varepsilon\kappa},$$

which equals the value at the portion $z > z^s$.

Shadow Boundary in Case $\beta = 0$. Next, we solve the case when regulators are oblivious to any information. Consider the set of values of $\{\lambda, z\}$ that guarantee the bank's solvency after a default event. We show next that these set of values satisfy

$$\lambda + J^\lambda(\lambda) \le \min\{\kappa,\ \Xi + (\Xi - 1)\cdot(z + J^z(z,\lambda))\},$$

guarantee solvency after the shock. Recall that the value of $z + J^z(z,\lambda) = \frac{z+\varepsilon\lambda}{1-\varepsilon\lambda}$ and the value of $\lambda + J^\lambda(\lambda) = \lambda\frac{(1-\varepsilon)}{1-\varepsilon\lambda}$. For values of $\kappa \ge \Xi + (\Xi-1)(z + J^z(z,\lambda))$, we have that:

$$\lambda \frac{(1-\varepsilon)}{1-\varepsilon\lambda} \le \Xi + (\Xi - 1)\frac{z+\varepsilon\lambda}{1-\varepsilon\lambda} \to \lambda \le \Xi + (\Xi - 1) z,$$

thus verifying the case where the bank avoids liquidation under the regulatory constraint with $\beta = 0$. For values of $\kappa < \Xi + (\Xi - 1) J^z(z,\lambda)$, we have that:

$$\lambda \le \frac{\kappa}{1-\varepsilon+\varepsilon\kappa}.$$

This case corresponds to the market based constraint. Thus, any $\lambda \le \Lambda(z) \equiv \min\left\{\frac{\kappa}{1-\varepsilon+\varepsilon\kappa}, \Xi + (\Xi-1) z\right\}$ survives a default event.

Next, we compute the values of z such that the market-based liquidation boundary is smaller than the regulatory counterpart. That is, the values of z such that:

$$\kappa < \Xi + (\Xi - 1)(z + J^z(z, \Xi + (\Xi-1) z)).$$

In this case, we obtain:

$$\kappa < \Xi + (\Xi - 1) \frac{z + \varepsilon (\Xi + (\Xi - 1) z)}{1 - \varepsilon (\Xi + (\Xi - 1) z)}.$$

Define $x(z) \equiv \varepsilon (\Xi + (\Xi - 1) z)$, thus:

$$\begin{aligned}
\kappa &< \Xi + (\Xi - 1) \frac{z + x(z)}{1 - x(z)} \rightarrow \\
\kappa &< \frac{\Xi - \Xi x(z) + (\Xi - 1) z + (\Xi - 1) x(z)}{1 - x(z)} \rightarrow \\
\kappa &< \frac{\Xi + (\Xi - 1) z - x(z)}{1 - x(z)}. \\
\kappa &< \frac{x(z)/\varepsilon - x(z)}{1 - x(z)}
\end{aligned}$$

Re-arranging terms, we obtain:

$$\begin{aligned}
\frac{\kappa\varepsilon}{1 - \varepsilon} &< \frac{x(z)}{1 - x(z)} \rightarrow \\
x(z) &> \frac{\kappa\varepsilon}{1 - \varepsilon + \kappa\varepsilon}.
\end{aligned}$$

Replacing $x(z)$, we obtain:

$$\begin{aligned}
(\Xi - 1) z &> \frac{\kappa}{1 - \varepsilon + \kappa\varepsilon} - \Xi \rightarrow \\
z &> \frac{1}{(\Xi - 1)} \frac{\kappa - \Xi (1 - \varepsilon + \kappa\varepsilon)}{1 - \varepsilon + \kappa\varepsilon}.
\end{aligned}$$

Thus, for any $z > z^s$, the market-based liquidation is tighter, where:

$$z^s \equiv \frac{1}{\Xi - 1} \frac{\kappa - \Xi (1 - \varepsilon + \kappa\varepsilon)}{1 - \varepsilon + \kappa\varepsilon}.$$

Hence, we have that the shadow boundary is:

$$\Lambda(z) = \begin{cases} \frac{\kappa}{1 - \varepsilon + \varepsilon\kappa} & \text{if } z > z^s \\ \\ \Xi + (\Xi - 1) z & \text{if } z \leq z^s. \end{cases}$$

We verify the continuity of the shadow boundary at z^s:

$$\Lambda(z^s) = \Xi + (\Xi - 1) z^s = \frac{\kappa}{(1 + (\kappa - 1) \varepsilon)} = \frac{\kappa}{1 - \varepsilon + \varepsilon\kappa},$$

which equals the value at the portion $z > z^s$.

Any point at the shadow boundary of the regulatory constraint, either jumps to another point at the shadow boundary or to the market based constraint. This condition is intuitive: since the regulator never observes a loss when $\beta = 0$, any shock that starts at the regulatory limit should puts the bank at the regulator limit because neither book loans nor book equity change with the shock.

E.2 Immediate Accounting Characterization: Proof of Propositions 1 and Corollary 1

The proof of the first part of Proposition 1 follows a special case of the proof of Proposition 2 which, in turn, is further proved for general preferences on bank dividends.

Main Result. In this Appendix we prove the following result that characterizes the solution to the bank's problem under immediate accounting. We specialize to linear objectives and exogenous dividends in the body of the paper:

Proposition 5 *[Bank's Problem] The bank's value function when $\alpha \to \infty$ is*

$$0 = \max_{\{c\}} \underbrace{f(c, v^*)}_{\text{dividend choice}} + v^* \cdot (\Omega^* - c) \tag{49}$$

where Ω^* is the maximal expected leveraged bank return,

$$\Omega^* = r^D + \underbrace{\max_{\lambda \in [1, \min\{\Xi, \kappa\}]} \underbrace{\left(r^L - r^D\right)}_{\text{levered return}} \lambda + \sigma \left\{(1 - \varepsilon\lambda)\, \mathbb{I}_{[\lambda \leq \Lambda]} + v^o \mathbb{I}_{[\lambda > \Lambda]} - 1\right\}}_{\text{leverage choice}}.$$

Derivation of the Main Result. Under immediate accounting, $v_z = 0$, $v_z \mu_z = 0$ in Proposition 2. If $\lambda > \kappa$, the bank is liquidated immediately and its value is $v^o < \bar{c}/\rho$, which is suboptimal. Thus, the problem in Proposition 2 simplifies to:

$$0 = \max_{\{c\}} f(c, v^*) + v^* \mu^W + \sigma \left\{v^* \left[1 + J^w \mathbb{I}_{[\lambda \leq \Lambda]}\right] + v^o \mathbb{I}_{[\lambda \in (\Lambda, \kappa]]} - 1\right\}.$$

Thus, replacing μ^W, we can write:

$$0 = \max_{\{c\}} f(c, v^*) + v^* \cdot (\Omega^* - c)$$

where:

$$\Omega^* = r^D + \max_{\lambda \in [1, \kappa]} \underbrace{\left(r^L - r^D\right)}_{\text{levered return}} \lambda + \sigma \left\{ J \mathbb{I}_{[\lambda \leq \Lambda]} + \left(\frac{v^o}{v^*} - 1\right) \mathbb{I}_{[\lambda > \Lambda]} \right\}$$

and

$$J \equiv -\varepsilon\lambda.$$

Specializing to the case of constant dividends and using the limit representation in (41), the objective simplifies to:

$$f(c, v^*) \to \bar{c} - v^* \rho,$$

where $\bar{c}$ is any constant rate. This corresponds to the special case in Proposition 1. Re-arranging terms, in the constant dividend case we obtain:

$$v^* = \frac{\bar{c}}{\rho - \Omega^*}.$$

Proving Corollary 1. We prove the result first for $\kappa = \min\{\Xi, \kappa\}$ and then generalize. Since

$$r^L - r^D > \sigma\varepsilon > 0,$$

the objective is piece-wise linear in λ:

- For any $\lambda < \Lambda$, the objective in Ω^* increases with leverage linearly with slope $r^L - r^D - \sigma\varepsilon$. Thus, setting $\lambda = \Lambda$ is optimal within $\lambda \in [1, \Lambda]$.
- For any, $\lambda \in (\Lambda, \kappa]$, the objective in Ω^* increases with leverage linearly with slope $r^L - r^D$. Thus, setting $\lambda = \kappa$ is optimal within $\lambda \in (\Lambda, \kappa]$.

The objective as a negative discontinuity at $\lambda = \Lambda$. Thus, without loss of generality, the bank must choose between two values $\lambda = \{\Lambda, \kappa\}$. Thus, we obtain:

$$\Omega^* = r^D + \max\left\{\left(r^L - r^D\right)\kappa + \sigma\left(\frac{v^o}{v^*} - 1\right), \quad \Lambda\left(\left(r^L - r^D\right) - \varepsilon\sigma\right)\right\}.$$

Setting leverage at the liquidation boundary is optimal if:

$$\left(r^L - r^D\right)\kappa - \sigma\left(1 - \frac{v^o}{v^*}\right) > \frac{\kappa}{1 + \varepsilon\left(\kappa - 1\right)}\left(r^L - r^D - \varepsilon\sigma\right).$$

This is the condition presented in the statement of Corollary 1. Next, we solve for the values of κ such that the condition above holds.

We express this inequality in terms of a quadratic equation. First, multiplying both sides by the denominator in the right:

$$\left(r^L - r^D\right)\kappa - \sigma\left(1 - \frac{v^o}{v^*}\right) + \varepsilon\left(\kappa - 1\right)\left(\left(r^L - r^D\right)\kappa - \sigma\left(1 - \frac{v^o}{v^*}\right)\right) > \kappa\left(r^L - r^D - \varepsilon\sigma\right)$$

Clearing terms:

$$-\sigma\left(1 - \frac{v^o}{v^*}\right) + \varepsilon\left(\kappa - 1\right)\left(\left(r^L - r^D\right)\kappa - \sigma\left(1 - \frac{v^o}{v^*}\right)\right) > -\varepsilon\kappa\sigma.$$

Reorganizing this expression into its polynomial components, we obtain:

$$\kappa^2\varepsilon\left(r^L - r^D\right) + \kappa\varepsilon\left(\sigma\frac{v^o}{v^*} - \left(r^L - r^D\right)\right) - \left(1 - \varepsilon\right)\sigma\left(1 - \frac{v^o}{v^*}\right) > 0.$$

Next, we solve for the critical roots:

$$\begin{aligned}
\kappa^o &= \frac{\varepsilon\left(\left(r^L - r^D\right) - \sigma\frac{v^o}{v^*}\right) \pm \sqrt{\varepsilon^2\left(r^L - r^D - \sigma\frac{v^o}{v^*}\right)^2 + 4\varepsilon\left(r^L - r^D\right)\left(1 - \varepsilon\right)\sigma\left(1 - \frac{v^o}{v^*}\right)}}{2\varepsilon\left(r^L - r^D\right)} \\
&= \frac{1}{2}\left(1 - \frac{\sigma}{\left(r^L - r^D\right)}\frac{v^o}{v^*} \pm \sqrt{\left(1 - \frac{\sigma}{\left(r^L - r^D\right)}\frac{v^o}{v^*}\right)^2 + 4\frac{\left(1 - \varepsilon\right)}{\varepsilon}\frac{\sigma}{\left(r^L - r^D\right)}\left(1 - \frac{v^o}{v^*}\right)}\right).
\end{aligned}$$

Since the discriminant is positive, because

$$4\varepsilon\left(r^L - r^D\right)\left(1 - \varepsilon\right)\sigma\left(1 - \frac{v^o}{v^*}\right) > 0,$$

there are two real roots, one negative and, at most, one positive. Only positive solutions are valid since $\kappa > 1$. The intercept is negative while the quadratic coefficient is positive. Thus, the bank risks liquidation if $\kappa > \kappa^o$. Hence, for any

$$\kappa > \kappa^o = \frac{1}{2}\left(1 - \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*} + \sqrt{\left(1 - \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}\right)^2 + 4\frac{(1-\varepsilon)}{\varepsilon}\frac{\sigma}{(r^L - r^D)}\left(1 - \frac{v^o}{v^*}\right)}\right) \tag{50}$$

the bank will risk liquidation setting leverage to the shadow boundary. This yields the value of λ_0 in Corollary 1. It's value is unique.

Next, we look for the combination of parameters such that $\kappa^o \geq 1$, so that the threshold is relevant. For such set of parameters, we have that the bank risks liquidation for any level of leverage constraints. The solution κ^o is less than 1 if

$$1 + \sqrt{\left(1 - \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}\right)^2 + 4(1-\varepsilon)\frac{\sigma}{\varepsilon(r^L - r^D)}\left(1 - \frac{v^o}{v^*}\right)} - \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*} \leq 2.$$

Re-arranging terms, the condition is true if:

$$\sqrt{\left(1 - \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}\right)^2 + 4(1-\varepsilon)\frac{\sigma}{\varepsilon(r^L - r^D)}\left(1 - \frac{v^o}{v^*}\right)} \leq 1 + \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}.$$

Since both sides are strictly positive, we have that:

$$\left(1 - \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}\right)^2 + 4(1-\varepsilon)\frac{\sigma}{\varepsilon(r^L - r^D)}\left(1 - \frac{v^o}{v^*}\right) \leq \left(1 + \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}\right)^2$$

$$4(1-\varepsilon)\frac{\sigma}{\varepsilon(r^L - r^D)}\left(1 - \frac{v^o}{v^*}\right) \leq \left(1 + \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}\right)^2 - \left(1 - \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}\right)^2 = 4\frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}.$$

Canceling terms, the condition becomes:

$$(1-\varepsilon)\left(1 - \frac{v^o}{v^*}\right) \leq \varepsilon\frac{v^o}{v^*} \to \varepsilon \geq \left(1 - \frac{v^o}{v^*}\right).$$

This guarantees that the threshold κ^o is greater than 1. Thus, λ^o is unique and greater than 1.

We finally generalize to the case where $\Xi = \min\{\Xi, \kappa\}$. Note that the threshold λ^o is independent of κ. The optimal leverage decision is thus:

$$\lambda = \min\{\Xi, \kappa\} \quad \text{if} \quad \min\{\Xi, \kappa\} > \lambda^o,$$

and

$$\lambda = \frac{\min\{\Xi, \kappa\}}{1 + (\min\{\Xi, \kappa\} - 1)\varepsilon} \quad \text{if} \quad \min\{\Xi, \kappa\} \leq \lambda^o.$$

E.3 Proof of Proposition 2

In this Appendix we prove the following general version of version of Proposition 2 where dividends are endogenous. We use this solution in the quantitative section. The solution is identical for the case of exogenous dividends. The general version of the proposition is the following.

Proposition 6 *[Bank's Problem] Given $\{z\}$, $V(Z,W) = v(z)W$, where v is the solution to the following HJB equation:*

$$0 = \max_{\{c,\lambda\}} f(c,v) + v_z\mu^z + v\mu^W + \sigma J^v \tag{51}$$

where J^v is the jump in the bank's value given a default event:

$$J^v = \left(\underbrace{\left(v\left(z+J^v\right)\left(1+J^W\right)-v(z)\right)}_{\text{jump in wealth}} \cdot \mathbb{I}_{[\lambda\leq\Lambda(z)]} + \underbrace{\left[v^o - v\right]}_{\text{liquiditation}} \cdot \mathbb{I}_{[\lambda>\Lambda(z)]}\right).$$

The optimal policies are given by: $C(Z,W) = c(z)\cdot W$.

The market value satisfies $S(Z,W) \equiv s(z)\cdot W$, where s solves:

$$\rho^I s = c(z) + s_z\mu^z + s\mu^W + \sigma J^s, \tag{52}$$

where J^s is given by:

$$J^s = \left(\underbrace{\left(s\left(z+J^z\right)\left(1+J^W\right)-s\right)}_{\text{jump in wealth}} \mathbb{I}_{[\lambda\leq\Lambda(z)]} + \underbrace{\left[s^o - s\right]}_{\text{liquidation}} \cdot \mathbb{I}_{[\lambda>\Lambda(z)]}\right)$$

Finally, Tobin's Q is given by:

$$Q(z) = s(z) \times q(z,\lambda(z)). \tag{53}$$

We can re-arrange the terms in the objective and obtain a reformulation:

Corollary 3 *[Bank's Problem] Given $\{z\}$, $V(Z,W) = v(z)\cdot W$, where v is the solution to the following HJB equation:*

$$0 = \max_{\{c\}} f(c,v) - (v - v_z z)c - v_z\alpha z + (v - v_z z)\Omega^* \tag{54}$$

where the optimal portfolio is:

$$\Omega^* = r^D + \max_{\{\lambda\}} \left(r^L - r^D\right)\lambda + \frac{J^v}{v(z) - v_z(z)},$$

and where:

$$J^v \equiv \left(\left[v\left(z+J^z\right)(1-\varepsilon\lambda) - v(z)\right]\cdot \mathbb{I}_{[\lambda\leq\Lambda(z)]} + \left(v^o - v(z)\right)\cdot \mathbb{I}_{[\lambda>\Lambda(z)]}\right)W.$$

Further specializing to the version with constant dividends yields Proposition 1 in the body of the text:

Proposition 7 *[Bank's Problem] Consider the problem of the bank with exogenous dividends, c. Given $\{z\}$, $V(Z,W) = v(z)\cdot W$, where v is the solution to the following HJB equation:*

$$0 = c + (v + v_z)\mu_z + (v - v_z z)\max_{\lambda}\left(\mu^W + \frac{J^v}{v - v_z}\right) \tag{55}$$

where

$$J^v \equiv \left(\left[v\left(z+J^z\right)(1-\varepsilon\lambda) - v(z)\right]\cdot \mathbb{I}_{[\lambda\leq\Lambda(z)]} + \left(v^o - v(z)\right)\cdot \mathbb{I}_{[\lambda>\Lambda(z)]}\right)W.$$

This condition is identical to the one that appears in the body of the paper. To prove the result, we present a formulation and then guess and verify the solution.

Formulation. Next, we prove Proposition 6. The primitive bank HJB equation, (10), is:

$$0 = \max_{\{C,\lambda\}} f\left(C, V\left(Z, W\right)\right) + \frac{E\left[dV\left(Z, W\right)\right]}{dt}. \tag{56}$$

Using a standard result in stochastic calculus for Jump processes:

$$\frac{E\left[dV\left(Z, W\right)\right]}{dt} = V_Z\left(Z, W\right)\mu^Z W + V_W\left(Z, W\right)\mu^W W + \sigma\mathbb{E}\left[J^V\right],$$

where J^V is given by:

$$J^V = \left[V\left(Z + J^Z, W + J^W\right) - V\left(Z, W\right)\right]\hat{\mathbb{I}} + \left(v^o W - V\left(Z, W\right)\right)\left(1 - \hat{\mathbb{I}}\right)$$

where

$$\hat{\mathbb{I}} = \begin{cases} 1 & \text{if } \lambda \leq \Lambda\left(Z/W\right) \\ 0 & \text{otherwise.} \end{cases}$$

Conjecture. We conjecture a solution to the value function and verify that it satisfies the HJB equation. The conjecture is:

$$V\left(Z, W\right) = v\left(z\right)W, \tag{57}$$

for a suitable candidate $v\left(z\right)$. Under this conjecture, we verify that $C\left(Z, W\right) = c\left(z\right) \cdot W$.

Factorization. We perform some useful calculations on the guess (57). In particular, we factorize equity from every term in the HJB equation. Under the conjecture,

$$\begin{aligned} f\left(C, V\right) &= f\left(c\left(z\right)W, v\left(z\right)W\right) \\ &= \frac{\rho}{1-\theta}v\left(z\right)W\left[\frac{c\left(z\right)^{1-\theta}W^{1-\theta}}{\left(v\left(z\right)W\right)^{1-\theta}} - 1\right] \\ &= \frac{\rho}{1-\theta}v\left(z\right)W\left[\frac{c\left(z\right)^{1-\theta}}{v\left(z\right)^{1-\theta}} - 1\right] \\ &= f\left(c\left(z\right), v\left(z\right)\right)W. \end{aligned} \tag{58}$$

The change in the value function with respect to zombie loans is:

$$\begin{aligned} V_Z &= \partial\left[v\left(Z/W\right)W\right]/\partial Z \\ &= v_z. \end{aligned}$$

The derivative of the value function with respect with respect to W is given by:

$$\begin{aligned} V_W &= \partial\left[v\left(Z/W\right)W\right]/\partial W \\ &= -v_z\frac{Z}{W} + v\left(z\right). \end{aligned} \tag{59}$$

Next, we collect terms to construct a modified drift for the value function:

$$\begin{aligned} V_Z \mu^Z W + V_W \mu^W W &= v_z(z) \cdot \left(-\alpha \frac{Z}{W}\right) W + \left(-v_z(z) \frac{Z}{W} + v(z)\right) \mu^W W \\ &= -v_z(z) \left(\alpha + \mu^W\right) zW + v(z) \cdot \mu^W W \\ &= \left(v_z(z) \mu^z + v(z) \cdot \mu^W\right) W. \end{aligned}$$

Finally, under the conjectured solution, the jump in the value function after an unrecognized default event is:

$$\begin{aligned} J^V &= \left[v\left(\frac{Z + J^Z}{W + J^W}\right)\left(W + J^W\right) - v(z) W\right] \hat{\mathbb{I}} + (v^o - v(z)) W \left[1 - \hat{\mathbb{I}}\right] \\ &= \left[v(z + J^z)(1 - \varepsilon\lambda) W - v(z) W\right] \hat{\mathbb{I}} + (v^o - v(z)) \left[1 - \hat{\mathbb{I}}\right] W \\ &= \left(\left[v(z + J^z)(1 - \varepsilon\lambda) - v(z)\right] \hat{\mathbb{I}} + (v^o - v(z)) \left[1 - \hat{\mathbb{I}}\right]\right) W, \\ &= \left(\left[v(z + J^z)(1 - \varepsilon\lambda) - v(z)\right] \hat{\mathbb{I}} + (v^o - v(z)) \left[1 - \hat{\mathbb{I}}\right]\right) W. \end{aligned}$$

Verification. We verify that the conjecture satisfies its HJB equation. We need to combine the pieces together. With the factorization above, (56) can be written as:

$$\begin{aligned} 0 &= \max_{\{c,\lambda\}} f(c, v) W \ldots \\ &+ \underbrace{\left[\begin{array}{cc} v_z(z) & v(z) \end{array}\right] \times \left[\begin{array}{c} \mu^z \\ \mu^W \end{array}\right]}_{\equiv \mu^v} \cdot W \ldots \\ &+ \sigma J^V W. \end{aligned}$$

where we used the fact that any choice of C can be expressed as a choice of $c(z) W$ as there is a one to one map from the $\{z, W\}$ space to the original space—by change of coordinates. Then, we can factor wealth from this HJB equation to express it as:

$$0 = \left[\max_{\{c,\lambda\}} f(c, v) + \mu^v + J^v\right] \cdot W,$$

and since the maximization is independent of W, this verifies the conjecture. Hence we have the proof of Proposition 6.

Collecting terms the solution to the HJB equation:

$$0 = \max_{\{c\}} f(c, v) - (v - v_z z) c - \alpha v_z z + (v - v_z z) \Omega(z), \tag{60}$$

where

$$\Omega(z) = r^D + \max_{\lambda \in [1, \Gamma(z)]} \left(r^L - r^D\right) \lambda - \sigma \frac{J^v}{v - v_z z}.$$

we verify the conjecture that the formula (57) satisfies the HJB equation (51). The factorization is valid as long as $v(z) - v_z(z) z > 0$. This is true since:

$$0 < V_W = -v_z(z) \frac{Z}{W} \frac{W}{W} + v(z) = v(z) - v_z(z) z.$$

This proves Proposition 3.

Finally, when the dividend is set to a constant rate and $\theta \to 0$, the value function specializes to:

$$0 = \rho c - \rho v - (v - v_z z)\, c - \alpha v_z z + (v - v_z z)\, \Omega(z), \tag{61}$$

where

$$\Omega(z) = r^D + \max_{\lambda \in [1, \Gamma(z)]} \left(r^L - r^D\right) \lambda - \sigma \frac{J^v}{v - v_z z}.$$

Since, the factor ρ is a monotone transformation of utility, this proves Proposition 3.

E.4 Proof of Corollary 2

We derive the first-order conditions of this problem. The optimal leverage is independent of dividends. We thus optimize this problem on its own. The next steps show that the solution is indeed bang-bang.

Optimal Leverage. Assume that indeed the value function $v(z)$ is concave. Consider the optimal leverage choice given by:

$$\Omega(z) = r^D + \max_{\lambda \in [1, \Gamma(z)]} \Omega^o(z, \lambda),$$

where

$$\Omega^o(z, \lambda) \equiv r^D + \left(r^L - r^D\right) \lambda + \sigma \left\{ \frac{\left[v\left(z + J^z\right)(1 - \varepsilon \lambda)\right] - v(z)}{v(z) - v_z(z)\, z} \right\}.$$

We now investigate the solution to the optimal λ.

Region $\lambda < \Lambda(z)$. The derivate of the objective with respect to λ is:

$$\frac{\partial \Omega^o(z, \lambda)}{\partial \lambda} \equiv \left(r^L - r^D\right) + \left[\frac{(1 - \lambda \varepsilon)\, v_z\left(z + J^z(\lambda, z)\right) J^z_\lambda(\lambda, z) - \varepsilon v\left(z + J^z(\lambda, z)\right)}{v(z) - v_z(z)\, z} \right]. \tag{62}$$

Recall that the jump of the zombie ratio is:

$$J^z(\lambda, z) = \varepsilon \lambda \left(\frac{z + 1}{1 - \varepsilon \lambda} \right).$$

Therefore. the derivative of the jump with respect to λ is:

$$\begin{aligned} J^z_\lambda(\lambda, z) &= J^z(\lambda, z) \left(\frac{1}{\lambda} + \frac{\varepsilon}{1 - \lambda \varepsilon} \right) \\ &= J^z(\lambda, z) \left(\frac{1}{\lambda} \cdot \frac{1}{1 - \varepsilon \lambda} \right) \end{aligned}$$

and, thus,

$$(1 - \lambda \varepsilon)\, v_z\left(z + J^z(\lambda, z)\right) J^z_\lambda(\lambda, z) = \varepsilon v_z\left(z + J^z(\lambda, z)\right) \frac{1}{\lambda} J^z(\lambda, z).$$

Hence, substituting this last expression into (62), we obtain:

$$\begin{aligned}\frac{\partial\Omega^{o}(z,\lambda)}{\partial\lambda} &= \left(r^{L}-r^{D}\right)-\sigma\varepsilon\left(\frac{v\left(z+J^{z}(\lambda,z)\right)-v_{z}\left(z+J^{z}(\lambda,z)\right)\frac{1}{\varepsilon\lambda}J^{z}(\lambda,z)}{v(z)-v_{z}(z)z}\right)\\ &= \left(r^{L}-r^{D}\right)-\sigma\varepsilon\left(\frac{v\left(\frac{z+\varepsilon\lambda}{1-\varepsilon\lambda}\right)-v_{z}\left(\frac{z+\varepsilon\lambda}{1-\varepsilon\lambda}\right)\left(\frac{z+1}{1-\varepsilon\lambda}\right)}{v(z)-v_{z}(z)z}\right).\end{aligned}$$

The derivative $\partial\Omega^{o}(z,\lambda)/\partial\lambda$ is positive if:

$$\frac{\left(r^{L}-r^{D}\right)}{\sigma\varepsilon}>\left(\frac{v\left(\frac{z+\varepsilon\lambda}{1-\varepsilon\lambda}\right)-v_{z}\left(\frac{z+\varepsilon\lambda}{1-\varepsilon\lambda}\right)\left(\frac{z+1}{1-\varepsilon\lambda}\right)}{v(z)-v_{z}(z)z}\right). \tag{63}$$

Recall that by assumption: $\lambda<\kappa<\frac{1}{\varepsilon}$. Thus, the term on the right of (63) satisfies:

$$\left(\frac{v\left(\frac{z+\varepsilon\lambda}{1-\varepsilon\lambda}\right)-v_{z}\left(\frac{z+\varepsilon\lambda}{1-\varepsilon\lambda}\right)\left(\frac{z+\varepsilon\lambda}{1-\varepsilon\lambda}\right)}{v(z)-v_{z}(z)z}\right)>\left(\frac{v\left(z+J^{z}(\lambda,z)\right)-v_{z}\left(z+J^{z}(\lambda,z)\right)\left(z+J^{z}(\lambda,z)\right)}{v(z)-v_{z}(z)z}\right).$$

A sufficiently condition for (63) is to show that:

$$\frac{r^{L}-r^{D}}{\sigma\varepsilon}\geq\frac{v(z')-v_{z}(z')z'}{v(z)-v_{z}(z)z} \tag{64}$$

for any pair z',z. We find bounds for the ratio of the right. Recall that:

$$v(z)-v_{z}(z)z=\frac{\partial V(1,z)W}{\partial W}=\frac{\partial V(W,Z)}{\partial W}.$$

We can bound, for any z, :

$$\frac{\partial V(W,zW)}{\partial W}\leq\lim_{\Xi\to\kappa}\frac{\partial V(W,Z')}{\partial W}=\frac{c}{\rho-\left(r^{D}+\kappa\left(r^{L}-r^{D}-\sigma\varepsilon\right)-c\right)}. \tag{65}$$

The bound follows because the marginal value of equity is higher if there is no regulation. The second equality follows directly from Proposition 1.

Next, we bound the value from below:

$$\frac{\partial V(W,Z')}{\partial W}\geq\lim_{\alpha,\Xi,\to\infty}\frac{\partial V(W,Z')}{\partial W}=\lim_{\alpha,\Xi,\to\infty}v(z)-v_{z}(z)z=\frac{c}{\rho-r^{D}-c}. \tag{66}$$

The bound follows because the marginal value of wealth is lowest when leverage is no admissible, as happens when $\alpha,\Xi\to0$. The second equality follows directly from Proposition 1.

Combining the bounds, (65) and (66),

$$\frac{\rho-r^{D}-c}{\rho-\left(r^{D}+\kappa\left(r^{L}-r^{D}-\sigma\varepsilon\right)-c\right)}\geq\frac{v(z')-v_{z}(z')z'}{v(z)-v_{z}(z)z}.$$

Thus, a sufficient condition for (64), is to show that:

$$\frac{r^{L}-r^{D}}{\sigma\varepsilon}\geq\frac{\rho-r^{D}-c}{\rho-\left(r^{D}+\kappa\left(r^{L}-r^{D}-\sigma\varepsilon\right)-c\right)}.$$

Subtracting one from both sides and cancelling terms we obtain that this solution is guaranteed as long as:

$$\rho \geq \left(r^D + \kappa\left(r^L - r^D\right) - c\right).$$

This condition holds by assumption. Thus, if $\lambda < \Lambda(z)$, setting $\lambda = \Lambda(z)$ increases the bank's value. The bank must be at a corner.

Region $\lambda \in (\Lambda(z), \quad \Gamma(z))$. Let $\lambda > \Lambda(z)$. The derivative of the objective is:

$$\left(r^L - r^D\right) > 0.$$

Thus, if $\lambda > \Lambda(z)$, setting $\lambda = \Gamma(z)$ increases the bank's value.

Summary. Since leverage is either at $\lambda = \Gamma(z)$ or $\lambda = \Lambda(z)$, as shown above, the solution must be bang-bang. That is:

$$\Omega(z) = r^D + \max\left\{\Omega^\Gamma(z), \quad \Omega^\Lambda(z)\right\},$$

where

$$\Omega^\Gamma(z) \equiv \left(r^L - r^D\right)\Gamma(z) + \sigma\frac{v^o - v(z)}{v(z) - v_z(z)\,z}$$

and

$$\Omega^\Lambda(z) \equiv \left(r^L - r^D\right)\Lambda(z) + \sigma\frac{v\left(z + J^z\left(\Lambda(z), z\right)\right)\left(1 - \varepsilon\Lambda(z)\right) - v(z)}{v(z) - v_z(z)\,z}$$

Optimal Dividend. When dividends are chosen, we have that the first-order condition for dividends is given by:

$$f_c(c, v) = v - v_z z,$$

we can solve this to obtain:

$$c = \rho^{1/\theta}\left[\frac{v}{(v - v_z z)^{1/\theta}}\right]. \tag{67}$$

For the case of log utility, $\theta \to 1$, and we verify that $c = \rho$ if v is constant, as in the case of immediate accounting.

E.5 Proof of Proposition 3: Optimal Regulation with Immediate Accounting Case

We prove the result in two steps. First, showing the solution to the first-best and then the one of the second best.

First Best. In the first best, we directly choose λ, ignoring any regulatory liquidations, $\Xi \geq \kappa$. Hence, from Proposition 2, the social optimal value of λ under immediate accounting solves:

$$r^D - c + \max_{\lambda}\left(\left(r^L - r^D\right)\lambda - \sigma\varepsilon\lambda \times \mathbb{I}_{[\lambda \leq \Lambda]} - \sigma\left(\varepsilon + (1-\psi)(1-\varepsilon)\right)\lambda \times \mathbb{I}_{[\lambda > \Lambda]}\right).$$

By assumptions 1 and 2,

$$\left(r^L - r^D\right) - \sigma\varepsilon > 0 > \left(r^L - r^D\right) - \sigma\left(\varepsilon + (1-\psi)(1-\varepsilon)\right).$$

Thus, the optimal leverage is increasing up to Λ and then decreasing.

$$\lambda^{fb} = \Lambda = \kappa \left(1 + \varepsilon \left(\kappa - 1\right)\right)^{-1}.$$

Second Best. Recall the bank's optimal response to regulation, equation (13):

$$\lambda^* \left(\Xi, \kappa\right) = \max \left\{\kappa, \Xi\right\} \times \mathbb{I}_{[\max\{\kappa,\Xi\}>\lambda^o]} + \max \left\{\kappa, \Xi\right\} \left(1 + \varepsilon \left(\max \left\{\kappa, \Xi\right\} - 1\right)\right)^{-1} \times \mathbb{I}_{[\max\{\kappa,\Xi\}\leq\lambda^o]},$$

where λ^o is given by (12). From Proposition 2, the social optimal value of λ under immediate accounting solves:

$$r^d - c + \max_{\Xi} \left(\left(r^L - r^D\right) - \sigma\varepsilon \times \mathbb{I}_{[\lambda\leq\Lambda]} - \sigma \left(\varepsilon + \left(1 - \psi\right)\left(1 - \varepsilon\right)\right) \times \mathbb{I}_{[\lambda>\Lambda]}\right) \lambda^* \left(\Xi, \kappa\right),$$

as described in the Proposition.

We study two cases:

Case I. $\kappa \leq \lambda^o$. If $\kappa \leq \lambda^o$,then for any $\Xi \geq \kappa$, we obtain:

$$\lambda^* \left(\Xi, \kappa\right) = \kappa \left(1 + \varepsilon \left(\kappa - 1\right)\right)^{-1} = \lambda^{fb}.$$

In this case, the first-best can be implemented with laissez faire regulation.

Case II. $\kappa > \lambda^o$. If $\kappa > \lambda^o$ then for any $\Xi \geq \kappa$, we obtain:

$$\lambda^* \left(\Xi, \kappa\right) = \kappa > \lambda^{fb},$$

and the value of the objective is:

$$r^D - c + \left(\left(r^L - r^D\right) - \sigma \left(\varepsilon + \left(1 - \psi\right)\left(1 - \varepsilon\right)\right)\right) \kappa.$$

Case II.a. $\Xi \in \left(\lambda^o, \kappa\right]$. If $\Xi \in (\lambda^o, \kappa]$, then $\lambda^* \left(\Xi, \kappa\right) = \Xi > \Lambda = \Xi \left(1 + \varepsilon \left(\Xi - 1\right)\right)^{-1}$ and the value of the regulatory constraint is:

$$r^D - c + \max_{\Xi} \left(\left(r^L - r^D\right) - \sigma \left(\varepsilon + \left(1 - \psi\right)\left(1 - \varepsilon\right)\right)\right) \Xi.$$

By Assumption 2, the objective is decreasing in Ξ. Thus, $\Xi = \lambda^o$ in this region.

Case II.b. $\Xi \in [0, \lambda^o]$. If $\Xi \in [0, \lambda^o]$, then $\lambda^* \left(\Xi, \kappa\right) = \Lambda = \Xi \left(1 + \varepsilon \left(\Xi - 1\right)\right)^{-1}$, the value of the regulatory constraint is:

$$r^D - c + \max_{\Xi} \left(\left(r^L - r^D\right) - \sigma\varepsilon\right) \Xi \left(1 + \varepsilon \left(\Xi - 1\right)\right)^{-1}.$$

By Assumption 2, the objective is increasing. Thus, within the range $\Xi \in [0, \kappa]$, the local maximum is $\Xi = \lambda^o$.

There is a discontinuity at λ^o. We must chose between setting Ξ to the right or to the left of λ^o. We set it to the left:

$$\left(\left(r^L - r^D\right) - \sigma\varepsilon\right) \lambda^o \left(1 + \varepsilon \left(\lambda^o - 1\right)\right)^{-1} \geq \left(\left(r^L - r^D\right) - \sigma \left(\varepsilon + \left(1 - \psi\right)\left(1 - \varepsilon\right)\right)\right) \lambda^o,$$

The right-hand side is negative By Assumption 2. Hence, in a second best, we have that $\Xi = \lambda^o$.

Summary. We conclude that when $\kappa > \lambda^o$, regulation is needed and is second best as the social value is:

$$\Omega^{sb} = r^D - c + \left(\left(r^L - r^D\right) - \sigma\varepsilon\right)\lambda^o\left(1 + \varepsilon\left(\lambda^o - 1\right)\right)^{-1} < r^d - c + \left(\left(r^L - r^D\right) - \sigma\varepsilon\right)\lambda^o = \Omega^{fb}.$$

When $\kappa \leq \lambda^o$, regulation is not necessary and $\Xi = \kappa$ achieves the first best Ω^{fb}.

E.6 Model Version with Loan Adjustment Costs

In this section, we derive a version of the model with adjustment costs on loans,

$$\Phi\left(I,L\right)=I+\frac{\gamma}{2}\left(\frac{I}{L}-\delta\right)^{2}L.$$

We can factor out L and employing the definition of ι to obtain:

$$\begin{aligned}\Phi\left(I,L\right) &= \left(\iota+\delta+\frac{\gamma}{2}\iota^{2}\right)L\\ &= \Phi\left(\iota,1\right)L+\delta L.\end{aligned}$$

Thus, we can express the funding cost relative to equity as:

$$\Phi\left(I,L\right)/W=\left(\Phi\left(\iota,1\right)+\delta\right)\lambda, \tag{68}$$

which is a function independent of the bank's size and depends on leverage and the investment rate.

Observation 1: Derivations of Laws of Motion. Now consider a time interval of length Δ. The law of motion for fundamental loans satisfies:

$$L_{t+\Delta}=\left(1-\delta\Delta\right)L_{t}+I_{t}\Delta-\varepsilon L_{t}\left(N_{t+\Delta}-N_{t}\right),$$

with the interpretation that the first term is the non-maturing fraction of loans, the second are loan issuances, and the third are losses in a time interval. Taking $\Delta\to 0$, we obtain the following law of motion:

$$dL=\left(I-\delta L\right)dt-\varepsilon L dN.$$

We express this law of motion in terms of net-worth to obtain:

$$dL=\iota\lambda W dt-\varepsilon\lambda W dN. \tag{69}$$

To ease the notation, we define the growth rate of fundamental loans and the jump relative to net-worth:

$$\mu^{L}\equiv\iota\lambda \text{ and } J^{L}\equiv-\varepsilon\lambda.$$

Similarly, for deposits we have that:

$$D_{t+\Delta}=\left(1+r^{D}\Delta\right)D_{t}-\left(r^{L}\Delta+\delta\Delta\right)L_{t}+\Phi\left(I_{t},L_{t}\right)\Delta+C_{t}\Delta$$

with the interpretation that the first term is the increase in deposits that results from paying interest with deposits; the second term is the reduction in deposits by the interest and principal payments on outstanding loans; the third term is the increase in deposits as a result of loan issuances; and the final term is dividend payments, all paid with deposits. Taking $\Delta\to 0$, we obtain the following law of motion:

$$dD=\left[r^{D}D-\left(r^{L}+\delta\right)L+\Phi\left(I,L\right)+C\right]dt.$$

We express this law of motion in terms of wealth to obtain:

$$dD=\left[r^{D}\left(\lambda-1\right)-\left(r^{L}+\delta\right)\lambda+\left(\Phi\left(\iota,1\right)+\delta\right)\lambda+c\right]W dt. \tag{70}$$

We define the growth rate of deposits relative to net-worth:

$$\mu^{D} \equiv r^{D}(\lambda-1)-\left(r^{L}+\delta\right)\lambda+(\Phi(\iota,1)+\delta)\lambda+c.$$

The evolution of Z is identical.

Observation 2: growth independence. Next, we present the evolution of net-worth with adjustment costs:

$$\begin{aligned} dW &= dL-dD \\ &= \left[\underbrace{\left(r^{L}+\delta\right)\lambda-r^{D}(\lambda-1)}_{\text{levered returns}}+\underbrace{(\iota-(\Phi(\iota,1)+\delta))\lambda}_{\text{capital loss from adjustment}}-\underbrace{c}_{\text{dividend rate}}\right]Wdt \\ &= \underbrace{-\varepsilon\lambda}_{\text{loss rate}}\ WdN. \end{aligned} \tag{71}$$

where the second line uses the laws of motion in (69) and (70), and employed observation 1.

F Model Appendix: Parametrization

This appendix section describes our calibration and estimation procedures in Section 4 in more detail. We use quarterly data from 1990 Q3 to 2021 Q1 to produce the target moments. Model moments are therefore also at quarterly frequency. To produce model moment counterparts, for each parameter draw, we simulate a panel of 10,000 banks for the same number of quarters as the data. Each sample simulation starts from the model's analytic stationary distribution, which we obtain by solving the Kolmogorov-Forward equation. From each sample, we calculate the cross-sectional average moments and construct the impulse-response functions (IRF).

F.1 Matching Facts

We estimate a more flexible version of the model presented in Section 3, by allowing for an endogenous dividend rate choice and assuming Duffie-Epstein preferences (rather than linear preferences) for the banker. The banker's risk aversion parameter is denoted as ψ and the intertemporal elasticity of substitution as $1/\theta$. To keep the parametrization tractable, we calibrate $\left\{r^L, r^D, \Xi\right\}$ independently, matching model moments to target moments in the data. Then, conditional on these calibrated parameters, we jointly estimate $\left\{\rho, \rho^I, \theta, \varepsilon, \alpha, \kappa\right\}$ using simulated method of moments together with the calibration of σ and v_0. The parameter values are listed in Table 1 in the main text. Table 8 presents both the targeted and untargeted moments in the data and the corresponding model moment.

Calibrated parameters. The exogenous returns on loans and deposits, r^L and r^D, are respectively set to 1.01% and 0.51%, consistent with the quarterly yield on loans (total interest income on loans divided by total loans) and the rate banks pay on their debt (total interest expenses divided by interest-bearing liabilities) in bank call reports. These values are also consistent with the calibration in Corbae and D'Erasmo (2021). We set the capital requirement parameter Ξ to 12.5, reflecting a Tier-1 risk-based capital ratio requirement of 8% at which a bank is considered well capitalized.[60]

Model Fit and Interpretation. Table 8 compares the moments generated by the model and those obtained from the data: our model fits most data moments well, with the exceptions of log market returns (which in the data includes other aggregate factors), the growth rate of book equity, and the common dividend rate. The model fits market leverage (8.596 in the data vs 8.274 in the model), book leverage (11.361 in the data vs 11.098 in the model), the log market return (2% in the data and 3.4% in the model), the market to book equity ratio (exact fit at 1.316), and the net charge-off rate (0.1% in the model vs 0.1% in the data) very tightly. Note that the capital requirement constraint limits banks' book leverage ratio to at most 12.5. Hence, in our model banks keep an equity buffer over the capital requirement constraint as in the data. [61] The model overshoots the dividend rate (0.6% in the data vs 3% in the model) and undershoots the growth rate of book equity (2% in the data vs. 0.4% in the model). In the data, banks can also repurchase shares to return cash to their shareholders, leading to a higher dividend rate in the model.

Table 8 presents unobservable model variables, such as fundamental leverage λ and q (fundamental equity/accounting value of equity). Fundamental leverage is 16.5, substantially higher than the book leverage value of 11.1. The average value for $q = W/\bar{W}$ is 0.69, implying that the fundamental value and the accounting value of equity differ by 31 percent. In terms of loans, zombie loans represent 3% of the total loans of banks.

[60] See the Federal Reserve Supervision and Regulation Report of November 2018, available here.

[61] In Appendix G, Figure G.1 presents the stationary distribution of fundamental leverage λ and the zombie loan to equity ratio z together with the liquidation set. It also shows that banks keep an equity buffer over the liquidation boundary determined by the regulatory constraint.

Table 8: Model and Data Moments

	Data	Model
Log Market Returns	0.020	0.034
	(0.176)	(0.032)
Market leverage	8.596	8.274
	(0.590)	(0.823)
Book Leverage	11.361	11.098
	(0.361)	(0.132)
Market to Book Equity	1.316	1.316
	(0.545)	(0.131)
Growth Rate of Book Equity	0.020	0.004
	(0.112)	(0.003)
Log Common Dividend Rate	0.006	0.030
	(0.006)	(0.001)
Charge-Off Rate	0.001	0.001
	(0.003)	(0.001)

	mean/(s.e.)
q	0.687
	(0.144)
z	0.540
	(0.438)
Zombie Loans/Total Loans	0.029
	(0.014)
λ	16.544
	(4.358)
c	0.061
	(0.009)
dW/W	0.004
	(0.071)
s	1.972
	(0.283)

Notes: The data uses the full sample from 1990 Q3 to 2021 Q1. The moments from the model are generated from a panel of 10,000 banks with the same number of quarters as in the data. The first row for each variable shows the mean. The second row shows the standard error of the mean in parenthesis. For market leverage, book leverage and market-to-book equity, the mean and standard error are computed on the logs, but when reporting the mean we apply exponential to show the mean in levels. All rates are quarterly.

G Model Appendix: Numerical Solution

We solve the model using the finite-differences method with an upwind scheme for the choice of forward or backward differences. Specifically, we compute the numerical derivatives of the value function $v(z)$ using finite differences and use the first order conditions to solve for c, and iterate on the HJB equation.

Our model is simple to solve because prices are constant, but presents two complications:

1. The size of the jump depends on the endogenous state variable. Starting from a point z, upon receiving a Poisson shock, the bank jumps to $z + J^z$. We use linear interpolation to get the value function off the grid.

2. The bank must choose which boundary to operate on. Rather than using a linear complementarity problem solver, we instead follow a relaxation approach. We assume that the choice of λ is taken infrequently: the bank can only switch from one boundary to the other upon arrival of an exogenous Poisson process with arrival rate $\mathcal{P}$. We take $\mathcal{P}$ to be large enough so that none of the results are affected by this approximation. In the limit, as $\mathcal{P} \to \infty$, the value function that we get converges to the desired one.

Let $b \in \{S, L\}$ denote the choice of the bank to operate in the shadow (S) or liquidation (L) boundary, so that:

$$b = \begin{cases} S & \text{if } \lambda = \Lambda(z) \\ L & \text{if } \lambda = \Gamma(z) \end{cases}$$

The HJB of the bank is:

$$\begin{aligned} 0 = \max_c f\left(c, v\left(z, S\right)\right) + \mu^z\left(z, \Lambda\left(z\right)\right) v_z\left(z, S\right) + \mu^W\left(\Lambda\left(z\right)\right) v\left(z, S\right) \\ + \sigma J^v\left(z, S\right) + \mathcal{P} \max\left\{v\left(z, L\right) - v\left(z, S\right), 0\right\} \end{aligned}$$

$$\begin{aligned} 0 = \max_c f\left(c, v\left(z, L\right)\right) + \mu^z\left(z, \Gamma\left(z\right)\right) v_z\left(z, L\right) + \mu^W\left(\Gamma\left(z\right)\right) v\left(z, L\right) \\ + \sigma J^v\left(z, L\right) + \mathcal{P} \max\left\{v\left(z, S\right) - v\left(z, L\right), 0\right\} \end{aligned}$$

where:

$$J^v\left(z, b\right) \equiv \begin{cases} v\left(z + J^z, S\right)\left(1 - \varepsilon\Lambda\left(z\right)\right) - v\left(z, S\right) & \text{if } b = S \\ v^o - v\left(z, L\right) & \text{if } b = L \end{cases}$$

Note that the value function $v\left(z, b\right)$ shows up non-linearly in the terms $f\left(c, v\left(z, b\right)\right)$—due to the assumption of Duffie-Esptein preferences—and in $\max\left\{v\left(z, L\right) - v\left(z, S\right), 0\right\}$. For these two terms we use the value function of the previous iteration. This circumvents the problem of having to solve for the value function non-linearly at every iteration, at the cost of having to choose a smaller time step when updating the value function—because the method used is semi-implicit.

The first order condition used for solving for the optimal policy $c\left(z\right)$ is:

$$f_c\left(c, v\left(z, b\right)\right) + z v_z\left(z, b\right) - v\left(z, b\right) = 0$$

where $f_c\left(c, v\right) = \beta(v/c)^\theta$.

G.1 Model Stationary distribution

To compute the stationary distribution, we use the Kolmogorov Forward equation. Let $g\left(z, b\right)$ be the invariant distribution over z conditional on choice of boundary b. Let $P_{b \to b'} = \mathcal{P} \cdot \mathbb{I}_{[v(z,b') > v(z,b)]}$

We have:

- in the shadow boundary:

$$\begin{aligned}
\dot{g}(z,S) &= -\frac{\partial}{\partial z}\left[\mu^{z}(z,S)\,g(z,S)\right] \\
&\quad -\sigma g(z,S)+\sigma\int_{0}^{\infty} g\left(z',S\right)\mathbb{I}_{[z'+J^{z}(z',S)=z]}dz' \\
&\quad -P_{S\to L}\cdot g(z,S)+P_{L\to S}\cdot g(z,L)
\end{aligned}$$

  for $z>0$. The first line represents changes in the density due to z drifting. The second line subtracts mass due to banks that had zombie loan ratio z and received a loan default shock, and adds mass from banks that after receiving a loan default shock have their zombie loan ratio jump to z. The third line computes changes in the density due to banks shifting boundaries. For $z=0$:

$$\begin{aligned}
\dot{g}(0,S) &= -\frac{\partial}{\partial z}\left[\mu^{z}(0,S)\,g(0,S)\right] \\
&\quad -\sigma g(0,S)+\sigma\int_{0}^{\infty} g(z,L)\,dz \\
&\quad -P_{S\to L}\cdot g(0,S)+P_{L\to S}\cdot g(0,L)
\end{aligned}$$

  which is different in the second line: we no longer have banks jumping to z after receiving a loan default shock (because $J^{z}>0$), but we add to the density to replace banks that were liquidated, to keep the mass of banks constant.

- in the liquidation boundary:

$$\begin{aligned}
\dot{g}(z,L) &= -\frac{\partial}{\partial z}\left[\mu^{z}(z,L)\,g(z,L)\right] \\
&\quad -\sigma g(z,L) \\
&\quad +P_{S\to L}\cdot g(z,L)-P_{L\to S}\cdot g(z,L)
\end{aligned}$$

for $z>0$. The first line represents changes in the density due to z drifting. The second line subtracts mass due to banks that had zombie loan ratio z and received a loan default shock. The third line computes changes in the density due to banks shifting boundaries.

The distribution of the state variable z and fundamental leverage λ is reported in Figure G.1. The grey area of the figure represents the liquidation region. The largest portion of the distribution lies in the shadow boundary.

G.2 Market returns

Let $S(z)$ be the valuation of a risk-neutral investor of the equity of a bank, which we use to construct market returns. It satisfies the following HJB equation:

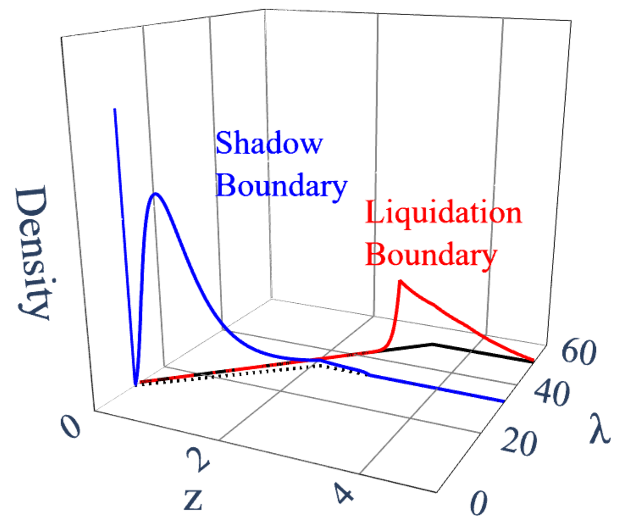
$$\rho^{I}S(z,W)=c(z)\,W+S_{Z}(Z,W)\,\mu^{Z}W+S_{W}(Z,W)\,\mu^{W}W+\sigma\left[S\left(Z+J^{Z},W+J^{W}\right)-S(Z,W)\right]$$

where $S(z)=0$ if $z>\Gamma(z)$. Just like $V(Z,W)$, $S(Z,W)$ is homogeneous in W and we can therefore express it as $S(Z,W)=s(z)\,W$, with $s(z)$ satisfying:

$$\rho^{I}s(z)=c(z)+s_{z}(z)\,\mu^{z}+s(z)\,\mu^{W}+\sigma\left[s\left(z+J^{z}\right)\left(1-\varepsilon\lambda(z)\right)-s(z)\right]$$

where $s(z)=0$ if $z>\Gamma(z)$.

Figure G.1: Model Stationary Distribution of Banks Across the z and λ State Space



Notes: This figure presents a two dimensional plot of the stationary distribution of banks across the (λ, z). The black dashed line traces out the shadow boundary $\Lambda(z)$ and the solid black line the liquidation boundary $\Gamma(z)$. The blue and red lines are the density of banks conditional on choosing the shadow and liquidation boundaries, respectively. The density conditional on choosing the liquidation boundary has been multiplied by 20 for visualization purposes, as it is otherwise not visible.

Using $s(z)$, we can construct market returns $r_t(z)$ as:

$$r_t(z) \equiv \frac{\int_{t-1}^{t} c_\tau(z) W_\tau d\tau + s_t(z) W_t}{s_{t-1}(z) W_{t-1}} \tag{72}$$

where t indexes quarters.

G.3 Formulas used to compute moments

We list here formulas that we derived to compute moments from the model.

At the bank level, we compute the bank failure rate as $\sigma \int_0^\infty g(z, L)\, dz$; book loans as $(\lambda + z) W$; the chargeoff rate as $\alpha(1 - \lambda/(z + \lambda))$; Tobin's Q as $s/(1+z)$; market equity as sW; liabilities as $(\lambda - 1) W$; market leverage as $(\lambda - 1)/s$; book loans over book equity as $(\lambda + z)/(1 + z)$; and distance to default as $\frac{1}{0.06}\frac{s+\lambda-1}{\lambda-1}$, where 0.06 stands for the volatility of market equity, which is common across banks.

At the aggregate level, the mean growth rate of equity is $\int_0^\infty \left[\mu^W(\Lambda(z)) - \sigma\varepsilon\Lambda(z)\right] g(z, S)\, dz + \int_0^\infty \left[\mu^W(\Gamma(z)) - \sigma\right] g(z, L)\, dz$. Note that, upon receiving a loan default shock, banks in the shadow boundary lose a fraction $\varepsilon\Lambda(z)$ of their equity and banks in the liquidation boundary lose 100% of their equity.

To aggregate variables to a quarterly frequency, we set time steps $dt = 1/30$ and for every 30 time steps we use the last value for stocks and the mean for flows.

G.3.1 Aggregate Loans

To compute the law of motion of loans L note that, in the shadow boundary, the drift of leverage is:

$$\lambda = \begin{cases} \frac{\Xi+(\Xi-1)z}{1+(\Xi-1)\varepsilon} & \text{if } z \leq z^s \\ \\ \frac{\kappa}{1+(\kappa-1)\varepsilon} & \text{if } z > z^s \end{cases} \Rightarrow \mu^{\lambda}_{Shd}(z) = \begin{cases} \frac{(\Xi-1)}{1+(\Xi-1)\varepsilon}\mu^z(z,S) & \text{if } z \leq z^s \\ \\ 0 & \text{if } z > z^s \end{cases}$$

and the jump in loans is:

$$J^L = \lambda' W' - L = \Lambda(z+J^z)(W-\varepsilon L) - L \Rightarrow \frac{J^L}{L} = \frac{\Lambda(z+J^z)}{\lambda}(1-\varepsilon\lambda) - 1$$

With these two terms, we can compute the expected loan growth rate of a bank in the shadow boundary as:

$$\frac{1}{dt}E\left[\frac{dL}{L}\right] = \frac{\mu^{\lambda}_{Shd}(z)}{\lambda} + \mu^W(\Lambda(z)) + \sigma\frac{J^L}{L}$$

Likewise, for a bank in the liquidation boundary:

$$\lambda = \begin{cases} \Xi + (\Xi-1)z & \text{if } z \leq z^m \\ \\ \kappa & \text{if } z > z^m \end{cases} \Rightarrow \mu^{\lambda}_{Liq}(z) = \begin{cases} (\Xi-1)\mu^z(z,L) & \text{if } z \leq z^m \\ \\ 0 & \text{if } z > z^m \end{cases}$$

$$J^L = 0 - L \Rightarrow \frac{J^L}{L} = -1$$

$$\frac{dL}{L} = \left(\frac{\mu^{\lambda}_{Liq}(z)}{\lambda} + \mu^W(\Gamma(z))\right)dt - 1dN$$

and hence:

$$\frac{1}{dt}E\left[\frac{dL}{L}\right] = \frac{\mu^{\lambda}_{Liq}(z)}{\lambda} + \mu^W(\Gamma(z)) - \sigma$$

We then compute the mean growth rate of loans as:

$$\int_0^{\infty}\left[\frac{\mu^{\lambda}_{Shd}(z)}{\lambda} + \mu^W(\Lambda(z)) + \sigma\frac{J^L}{L}\right]g(z,S)\,dz + \int_0^{\infty}\left[\frac{\mu^{\lambda}_{Liq}(z)}{\lambda} + \mu^W(\Gamma(z)) - \sigma\right]g(z,L)\,dz.$$

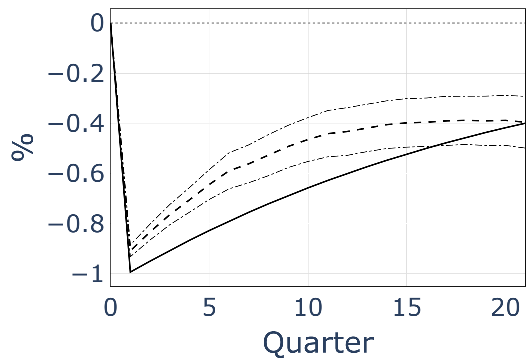
G.4 The Role of the IES for the Quantitative Fit

In Section 4.2 of the main text, we argue that the delayed loan loss recognition mechanism is driving the slow adjustment of banks to net-worth shocks. Since the bankers' preference imply an intertemporal smoothing incentive whenever $\theta > 0$, one might worry that instead what drives the slow adjustment is θ. We investigate the role of the IES ($1/\theta$) for the quantitative results by solving the model for the same parameter configuration as in the baseline model (see Section 4), except setting $\theta = 2$, and reestimate the IRF on the model generated data. This calibration implies significantly lower intertemporal smoothing incentives. The results are in Figure G.2.

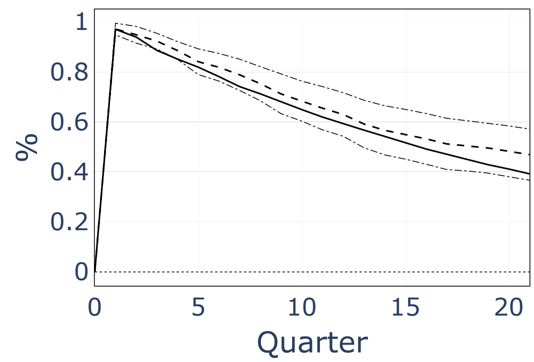
The blue line presents the impulse response functions of the model for the case when $\theta = 2$. Relative to Figure 10, the IRF still shows substantially slow adjustment to a negative net-worth shock.

Figure G.2: Data IRFs versus Model IRFs with $\theta = 2$

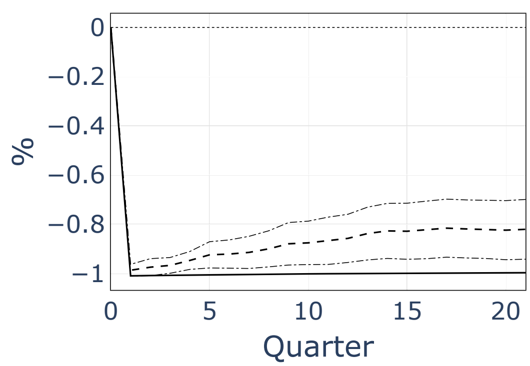
(a) Tobin's Q



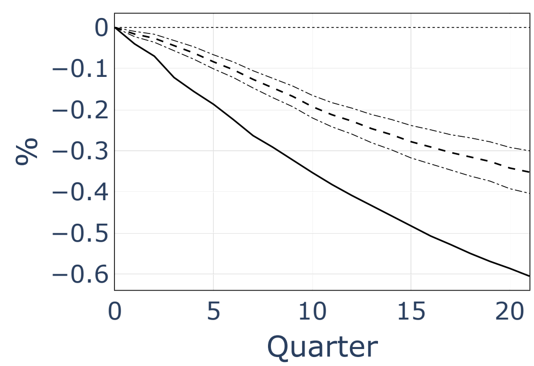
(b) Market Leverage



(c) Market Equity



(d) Liabilities



Notes: The figures present the impulse response functions of model simulated data (blue) for the benchmark calibration, except with the IES $1/\theta$ increased to $1/2$, and compares it to the data (gray line represents the point estimates and the shaded area the 95% confidence interval). We show the impulse response function of Tobin's Q (Panel (a)), market leverage (Panel (b)), market equity (Panel (c)), and liabilities (Panel (d)).

The higher IES makes liabilities adjust faster in response to a loan default shock, so the fit of the IRF for liabilites relative to the data worsens, and the one for market leverage improves, but the delayed adjustment feature remains.