# Financial Frictions with Risk, Irreversible Capital, and Default

Francisco Buera<sup>1,2</sup> Jeremy Majerovitz<sup>1,2</sup> Yongseok Shin<sup>1,2</sup> Kuldeep Singh<sup>1</sup>

<sup>1</sup>Washington University in St. Louis

<sup>2</sup>Federal Reserve Bank of St. Louis

June 30, 2023

The views expressed herein are those of the authors and do not necessarily represent those of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

#### Motivation

- Large cross-country income per-capita differences

- Credit frictions play a role

- Many models of credit frictions ignore investment risk, irreversibility

- Micro-development evidence: limited insurance > limited credit?
  - Udry (SED, 2012); Karlan, Osei, Osei-Akoto and Udry (2014)

## **Our Questions**

- Quantitative model of risk-averse entrepreneurs, irreversible investment, limited commitment/default
- Study how development outcomes are affected by the contractual environment
  - Relationship between economic and financial development?
  - Role of resalability frictions, collateral requirements?
  - Does a default option discourage/promote development?
  - Poverty trap?

## **Related Literature**

- Investment risk and capital accumulation: Angeletos (2007), Sandri (2014)
- Irreversible investment and misallocation: Asker, Collard-Wexler, De Loecker (2014); Boar, Gorea, Midrigan (2023)

- Entrepreneurial risk and default: Akyol and Athreya (2011); Morales (2022)
- Endogenous entrepreneurial risk: Vereshchagina and Hopenhayn (2009); Robinson (2023)

# Model Overview: Risk and Irreversibility

Start with entrepreneurs who invest under financial frictions

Standard model features strong "save-your-way-out" dynamics

- Productivity is known in a given period
- Rental market for capital subject to collateral constraint
- $\blacktriangleright \implies$  No investment risk
- ▶  $\implies$  Invest all the way up to the constraint, until  $r + \delta = f'(k)$

Here: risk-averse entrepreneurs make partially irreversible investments in capital, subject to the risk that their productivity may change in the future.

Entrepreneurs invest less in the firm because they are risk averse, so capital accumulation is slow.

(Under CRRA, entrepreneur eventually gets so rich that she is no longer risk averse in CARA sense.)

## Model Overview: Default

How does credit enter the picture?

Credit interacts with risk if the entrepreneur can default.

The option to default creates a state-contingent contract: if things get very bad, entrepreneur can default and not pay off debts.

Makes investment less risky. (Theoretically, may even overturn the underinvestment due to risk.)

Punishment for default is loss of capital and access to credit (temporary).

Zero profits for banks  $\implies$  default raises borrowers' interest rate

#### Entrepreneur's Problem

$$\max_{c_t, i_t, \tau} \mathbb{E}_0 \int_0^\tau e^{-\rho t} \frac{c_t^{1-\sigma}}{1-\sigma} dt + e^{-\rho \tau} V^{def}(0, 0, z_t)$$
$$da = \left(\pi(k_t, z_t) + w + r_a a_t - c_t - i_t - \Phi(i_t, k_t)\right) dt$$
$$dk = (i_t - \delta k_t) dt$$
$$a_t \ge -\lambda k_t, \quad k_t \ge 0$$
$$\pi(k_t, z_t) \equiv \max_l z_t k_t^\alpha l_t^\beta - wl$$
$$z_t \in \{z_1, z_2, z_3\}$$

• Borrow  $(a \leq 0)$  at rate  $r_a = r_b$ .

- Save (a > 0), at rate  $r_a = r_s$   $(r_s \le r_b)$ .
- Stochastic productivity transitions (Poisson)
- Can choose to default (optimal stopping time τ); gets V<sup>def</sup>(0,0,z<sub>t</sub>) (lose capital and credit access)

#### Entrepreneur's Problem in Default

$$\max_{c_t, i_t} \mathbb{E}_0 \int_0^T e^{-\rho t} \frac{c_t^{1-\sigma}}{1-\sigma} dt + e^{-\rho T} V(a_T, k_T, z_T)$$
$$da = \left(\pi(k_t, z_t) + w + r_s a_t - c_t - i_t - \Phi(i_t, k_t)\right) dt$$
$$dk = (i_t - \delta k_t) dt$$
$$a_t \ge 0, \quad k_t \ge 0$$

When the entrepreneur defaults:

- Debt goes to zero, but the bank takes all capital
- Cannot borrow until...
- ▶ Regain credit access with Poisson intensity  $\chi_{dn}$ .

#### Adjustment Costs and Partial Irreversibility

When the firm invests, needs to pay adjustment cost,  $\Phi(i,k),$  in addition to cost of investment.

When investment is negative (selling off capital), only gets back  $\phi \leq 1$  dollars for each dollar of capital sold.

Also pays small quadratic adjustment cost: this is just to make the problem smooth

$$\Phi(i,k) = \begin{cases} \frac{\kappa}{2} \left[ \frac{i}{k+\bar{k}} \right]^2 (k+\bar{k}) & i \ge 0\\ -(1-\phi)i + \frac{\kappa}{2} \left[ \frac{i}{k+\bar{k}} \right]^2 (k+\bar{k}) & i < 0 \end{cases}$$

# Value Functions

With credit access:

$$\rho V(a,k,z_j) = \max\left\{\rho V^{def}(0,0,z_j),\right.$$
$$\max_{c,i} \left[\frac{c^{1-\sigma}}{1-\sigma} + V_a \cdot \left(\pi(k,z_j) + w + r_a \cdot a - c - i - \Phi(i,k)\right) + V_k \cdot (i - \delta k) + \sum_{-j} \lambda_{z_{j,-j}} \left(V(a,k,z_{-j}) - V(a,k,z_j)\right)\right]\right\}$$

Without credit access:

$$\begin{split} \rho V^{def}(a,k,z_{j}) &= \\ \max_{c,i} \left[ \frac{c^{1-\sigma}}{1-\sigma} + V_{a}^{def} \cdot \left( \pi(k,z_{j}) + w + r_{a} \cdot a - c - i - \Phi(i,k) \right) \right. \\ &+ V_{k}^{def} \cdot (i - \delta k) + \sum_{-j} \lambda_{z_{j,-j}} \left( V^{def}(a,k,z_{-j}) - V^{def}(a,k,z_{j}) \right) \\ &+ \chi_{dn} \cdot \left( V(a,k,z_{j}) - V^{def}(a,k,z_{j}) \right) \right] \end{split}$$

## Bank's Problem

Bank lends at  $r_b$  with loan-to-value constraint (requires  $1/\lambda$  units of capital as collateral for each dollar of debt)

Same  $r_b$  for all borrowers: does not depend on (a, k, z). May result from information constraints and/or legal constraints.

If entrepreneur defaults, bank liquidates the firm and gets back  $\phi_b \cdot k$ . In the baseline model, bank has same liquidation technology as entrepreneur  $(\phi_b = \phi)$ .

Bank borrows at rate  $r_{s},\, {\rm perfectly}$  elastic supply. Makes zero profit in equilibrium.

#### Bank's Zero Profit Condition

Let G(a, k, z) be the joint c.d.f.

 $\Delta$  is a small time interval.

 $\mathcal{I}_{def}$  is the default region (changes with  $\Delta$ )

Let  $B:=\int_{a\leq 0}-adG(a,k,z)$  denote total debt.

Zero profits implies (discrete time approximation):

$$r_s B\Delta = r_b B\Delta + \int_{(a,k,z) \in \mathcal{I}_{def}} (\phi_b \cdot k - (-a)) dG(a,k,z)$$

Limit as  $\Delta \rightarrow 0$  (continuous time):

 $r_b = r_s + \text{Default Risk Premium}$ 

Default Risk Premium = 
$$\lim_{\Delta \to 0} \frac{\int_{(a,k,z) \in \mathcal{I}_{def}} (\phi_b \cdot k + a) dG(a,k,z)}{B\Delta}$$

# Planner's Problem (Static)

To measure deviations from efficiency, solve planner's problem.

Given distribution  $\Omega(k, z)$ , allocation of labor is a static problem:

$$\max_{l} \int zk^{\alpha} l^{\beta} d\Omega \quad s.t \quad \int l d\Omega = 1$$
$$l(k, z) = \frac{\left(zk^{\alpha}\right)^{\frac{1}{1-\beta}}}{\int \left(zk^{\alpha}\right)^{\frac{1}{1-\beta}} d\Omega}$$

However, planner must solve dynamic investment/liquidation problem...

## Planner's Problem (Dynamic)

Planner takes into account NPV of resource flows:

$$\max_{i_t} \int_0^\infty e^{-\rho t} \int \left( y(k_t, z_t) - i_t - \Phi(i_t, k_t) - MPL_t \cdot l(k_t, z_t) \right) d\Omega_t dt$$
$$dk = (i_t - \delta k_t) dt$$

where

$$y(k_t, z_t) = \frac{z_t^{\frac{1}{1-\beta}} k_t^{\frac{\alpha}{1-\beta}}}{\left[\int \left(zk^{\alpha}\right)^{\frac{1}{1-\beta}} d\Omega\right]^{\beta}}$$

Planner invests less than static optimum: takes into account costly liquidation due to negative productivity shocks.

## Parameters for Numerical Example

Parameters	Values
ho (discount rate)	0.083
$\sigma$ (risk aversion)	2
$r_s$ (saving rate)	0.02
$\delta$ (depreciation)	0.06
$\phi,\kappa,k$ (adjustment cost)	0.35,0.1,0.1
lpha,eta (production function)	0.3,0.49
$\chi_{dn}$ (regain credit access)	0.5
$z_1, z_2, z_3$ (productivity)	0,1.45,1.75
$\lambda$ (loan-to-value constraint)	0.75

#### Table: Productivity Process

	$z_1$	$z_2$	$z_3$	Stationary Mass
$z_1$	-	0.05	0.02	0.87
$z_2$	1	-	0.1	0.05
$z_3$	0.125	0.15	-	0.08

## **Decision Rules**



Figure: Investment

 $\mathsf{Red:} \ i > 0 \qquad \mathsf{Gray:} \ i = 0 \qquad \mathsf{Blue:} \ i < 0$ 

### **Decision Rules**



Figure: Saving

 $\mathsf{Red:}\ \dot{a} > 0 \qquad \mathsf{Gray:}\ \dot{a} = 0 \qquad \mathsf{Blue:}\ \dot{a} < 0$ 

## Phase Diagram

Figure:  $z = z_1$ 



#### Time Path and Default Basin

Figure:  $z = z_3$  Time Path



"Default Basin" for  $z = z_1$ : 1, 5, 25 yrs

# Role of Default Option

Variable	No Default	With De-	
		fault	
$r_b(borrowing rate)$	2%	26.4%	
Fraction without	0	0.031	
credit access			
Default rate	-	0.41	
a < 0	0.240	0.037	
$Y/Y^p_{\phi=0.35}$	0.85	0.826	
$K/K^p_{\phi=0.35}$	0.858	0.743	
$K_{z_1}/K_{z_1,\phi=0.35}^p$	1.107	0.886	
$K_{z_2}/K_{z_2,\phi=0.35}^p$	0.740	0.688	
$K_{z_3}/K^p_{z_3,\phi=0.35}$	0.616	0.604	
$TFP/TFP^p_{\phi=0.35}$	0.9	0.924	

- Less investment with default option in the long run
- Lower output in the long run but moderately better allocative efficiency (less capital by z<sub>1</sub>)

## Planner vs. Entrepreneur with Default Option



Figure: Stationary Density of k by z

Solid lines: Planner; Dashed lines: Benchmark with default option

#### Planner vs. Entrepreneur: Capital Paths

Figure:  $z = z_3$  starting with a = 0, k = 0



Because of risk, entrepreneurs accumulate more slowly than planner
Faster capital growth due to leverage (early on) and default option

#### Entrepreneur: Asset Paths

Figure:  $z = z_3$  starting with a = 0, k = 0



# Comparative Statics w.r.t. $\phi$

Variable	$\phi = 0.1$	$\phi = 0.27$	$\phi = 0.35$	$\phi = 0.75$
$r_b$	110%	42.8%	26.4%	2%
Fraction without	0.027	0.03	0.031	0.006
credit access				
Default Rate	0.78	0.47	0.41	0.021
a < 0	0.018	0.032	0.037	0.15
$Y/Y_{\phi=0.35}^{p}$	0.775	0.806	0.826	1.007
$K/K^p_{\phi=0.35}$	0.867	0.857	0.743	0.891
$K_{z_1}/K^p_{z_1,\phi=0.35}$	1.25	1.174	0.886	0.519
$K_{z_2}/K^p_{z_2,\phi=0.35}$	0.691	0.712	0.688	1.115
$K_{z_3}/K^p_{z_3,\phi=0.35}$	0.5	0.548	0.604	1.235
$TFP/TFP^p_{\phi=0.35}$	0.818	0.855	0.924	1.052
$K^p/K^p_{\phi=0.35}$	1.755	1.271	1	0.914

 $\phi$ : fraction remains after sale of capital ( $\phi_b$  for bank)

Less friction (higher  $\phi$ ) leads to in the long run

- More borrowing, less default
- More output, more investment by z<sub>3</sub>
- Better allocative efficiency

## Comparative Statics w.r.t. $\phi$

Figure: Borrowing Rate (%)



#### Comparative Statics w.r.t. $\phi$



All values normalized by the corresponding planner value with  $\phi = 0.35$ 

φ captures... technology, contractual frictions (e.g., asymmetric information), market thickness

Comparative Statics w.r.t.  $\lambda$ 

Figure: Borrowing Rate (%)



#### Comparative Statics w.r.t. $\lambda$



 $\triangleright$   $\lambda$  affects borrowing, but has muted effects on quantities (long run).

# Unpacking Comparative Statics w.r.t. $\phi$ and $\phi_b$

Variable	$\phi_b = 0.35$	$\phi_b = 0.35$	$\phi_b = 0.65$	$\phi_b = 0.65$
$\phi$	0.65	0.35	0.35	0.65
$r_b$	13.3%	26.4%	5.5%	4.4%
Without credit	0.027	0.031	0.034	0.028
access				
Default Rate	0.34	0.41	0.26	0.29
a < 0	0.038	0.037	0.065	0.048
$Y/Y^{p}_{\phi=0.35}$	0.918	0.826	0.86	0.936
$K/K_{\phi=0.35}^{p}$	0.693	0.743	0.86	0.73
$K_{z_1}/K_{z_1,\phi=0.35}^p$	0.435	0.886	1.047	0.453
$K_{z_2}/K_{z_2,\phi=0.35}^p$	0.85	0.688	0.79	0.894
$K_{z_3}/K^p_{z_3,\phi=0.35}$	0.934	0.604	0.676	0.997
$TFP/TFP^p_{\phi=0.35}$	1.055	0.924	0.911	1.055

 $\phi$ : fraction remains after sale of capital ( $\phi_b$  for bank)

•  $\phi$  has larger effects on quantities, but  $\phi_b$  has larger effects on borrowing and default (long run).

# Role of Default Option: Skiba (1978) Technology



Skiba Technology: Time Paths

Figure:  $z = z_3$ 



# Skiba Technology: Time Paths

Figure: 
$$z = z_3$$
 starting with  $a = 0$ ,  $k = 0$ 



- Unproductive at small scale, slower investment, followed by big jump financed by (defaultable) debt
- Without a default option, poverty trap

## Skiba Technology: Time Paths





# Taking Stock

- Uninsurable investment risk due to irreversibility can lead to significant underinvestment (more so than collateral constraint)
- The option to default can be an important insurance mechanism overcoming this, especially with non-convexproduction functions (e.g., high fixed cost)
- Work in progress: Differentiated loan contracts, richer/better quantification strategy...